An Effective Remedial Instruction in Number Sense for Third Graders in Taiwan

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Abstract

To examine the effect of remedial instruction on number sense performance, two third-grade classes from an elementary school in Taiwan joined a 7-week remedial instructional program (one number sense-based [NS-based] remedial instruction class and one textbook-based remedial instruction class). The results from both tests and interviews showed that students in the NS-based remedial instruction class made more progress in number sense than those in the textbook-based remedial instruction class. This implies that the NS-based remedial instruction had a more positive effect on students' number sense performance than the textbook-based instruction. This finding further indicates that NS-based remedial instruction via well-organized teaching materials and appropriate math manipulatives has a positive impact on students' number sense development. Some educational implications are also discussed.

Recent studies and international white papers have highlighted that the development of children's number sense should be incorporated into school mathematics teaching (Jordan, Glutting & Ramineni, 2010; National Research Council [NRC], 2001; National Council of Teachers of Mathematics [NCTM], 2000; Verschaffel, Greer, & De Corte, 2007). Due to its importance, more and more research studies around the world have focused on the teaching and learning of number sense (Griffin, 2004; Jordan, Kaplan, Oláh & Locuniak, 2006; Yang & Wu, 2010). However, current mathematics teaching in Taiwan continues to emphasize written computation for an exact answer; a practice that may obstruct students' development of number sense (Reys & Yang, 1998). In addition, Yang & Li (2008) also found that 5th-graders in Taiwan performed poorly on number sense and suggested that number sense should be taught at an earlier grade level. These results have encouraged researchers to consider the following questions. How has a student's number sense developed after completing the third grade mathematics course? Can textbook-based activities enhance students' number sense? If not, how can we improve students' number sense? Can we design and perform NS-based remedial instruction to promote students' number sense development? How much difference is there between NS-based remedial instruction and textbook-based remedial practice? Given these questions, the task of developing NS-based remedial instruction for third graders deserves attention by researchers in Taiwan. Accordingly, the following research questions were developed to address this issue:

(1) How can number sense activities be implemented in NS-based remedial instruction?
(2) Does number sense remedial instruction have a more positive impact on the development of students' number sense than textbook-based remedial practice?

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(3) Do students who receive number sense remedial instruction more frequently use number sense than students who receive textbook-based remedial practice?

Keywords: Number sense; remedial instruction; third graders

Background

Number sense is a way of thinking about and understanding numbers, operations, and the relationships between numbers and operations (McIntosh, Reys, & Reys, 1992; Sood & Jitendra, 2007; Yang & Li, 2008). Number sense also includes the ability to develop flexible and efficient strategies, such as mental computation and estimation, and to solve numerical problems including in daily life situations (Griffin, 2004; Yang & Wu, 2010). Why is the development of number sense so important? Earlier studies (Burns, 1994; Reys & Yang, 1998) showed that excessive reliance on rule-based methods to solve problems might limit and hinder children's mathematical thinking and understanding. Reys & Yang (1998) showed that children skilled in written computation did not necessarily develop better number sense. In addition, the over reliance on rule-based methods may lead students to produce many erroneous results (e.g., misapplication of calculations and formulas, misplacement of decimal points, or production of unreasonable results). However, the current emphasis on written computation in mathematics teaching in Taiwan may result in weakness or deficiency in number sense development. Moreover, there are few number sense learning activities oriented toward daily life designed to address third-grader's deficiencies in number sense. This study attempts to develop remedial instruction units to promote number sense that are based on children's misconceptions or deficient knowledge.

Theoretical Framework

Components of Number Sense

During the past two decades, several number sense studies and reports (Markovits & Sowder, 1994; McIntosh, et al., 1992; NCTM, 2000; Verschaffel et. al., 2007; Yang & Li, 2008) have proposed theoretical frameworks of number sense from different perspectives. No researchers in the field "define number sense in exactly the same way" (Berch, 2005, p. 333); however, they do include several common components. Based on these studies and reports, this study defines the components of number sense as follows:

(1) Understanding the meanings of numbers, operations and their relationships
This ability enables a student to make sense of numbers (whole numbers, fractions and decimals) and related operations (+, -, x, and ÷) and understand the relationship between numbers and operations (McIntosh et al., 1992; Yang & Tsai, 2010). For example, children know that 8 represents eight thousand in 28,036.

(2) Being able to use multiple representations of numbers and operations
This ability includes dealing with numbers and representing numbers in different ways (McIntosh et al., 1992; NCTM, 2000). For example, children should know $\frac{1}{5} = 0.2 = 20\%$.

(3) Recognizing the relative magnitude of numbers
This is an ability to compare and order numbers correctly (McIntosh et al., 1992; Yang & Tsai, 2010). For example, children should know 23/50 is less than 21/40 since 21/40 is more than 1/2 and that 23/50 is less than 1/2.

(4) Being able to compose and decompose numbers flexibly
This implies the ability to break down numbers for the convenience of computational fluency (McIntosh et al., 1992). For example, when students are asked to solve 24×25, they could

decompose the multiplicand 24 into 6×4 to get 6×4×25, and know it is equal to 6×100. This helps children solve problems efficiently.

(5) Judging the reasonableness of a computational result via different strategies
This concerns students' ability to utilize strategies such as estimation and mental computation to solve problems appropriately and to know if the result is reasonable (NCTM, 2000; Yang & Tsai, 2010). For example, children should be able to estimate the height of the classroom from the ceiling to the floor being about 3-4 meters.

Remedial Instruction

Earlier studies have suggested that an individual's use of number representations and mathematical thinking relied in part on the earlier development of number sense (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004). Students with a weak foundation of mathematical understanding often demand more time and attention, and require supplemental instruction to ensure later successful learning (Burns, 2007). Remedial instruction should be provided to ensure students' academic success (Finnish National Board of Education, 2009). Therefore, appropriate remedial instruction is necessary for those students who tend to have learning difficulties. Moreover, better development in number sense may prevent children from having trouble learning future mathematics content (Jordan, Kaplan, Locuniak & Ramineni, 2007; Sood & Jitendra, 2007; Verschaffel et al., 2007; Yang & Wu, 2010). At the same time, several studies have indicated that many students lack number sense (Alajmi & Reys, 2007; Menon, 2004; Reys & Yang, 1998; Yang & Li, 2008). This implies that an in-time remedial instruction focused on number sense is critical for primary school children who lack number sense.

Bottge, Rueda, Serlin, Hung and Kwon (2007) have suggested that the teaching practices used for students with special needs should adopt a constructivist approach. Yang (2006) also showed that adopting a constructivist approach for number sense teaching helped Taiwanese students develop better understanding for number sense. Therefore, the teaching practices in this study not only highlight small-group and whole-class discussion, but also encourage students to question, debate, and provide arguments to support their solutions during remedial instruction. This is different from the traditional textbook approach of emphasizing drills in prescribed procedures and deemphasizing explanation of students' solutions to problems (Bottge et al., 2007; Yang, 2006; Yang & Wu, 2010).

Given this discussion, it is reasonable to believe that remedial instruction should be a good approach to help children improve number sense. In the current study, two classes of third graders were selected to receive remedial instruction. The goal of remedial instruction was to compare whether NS-based remedial instruction or textbook-based remedial instruction had a more positive impact on students' number sense performance.

Method

Sample

Two third-grade classes (a NS-based remedial instruction class and a textbook-based remedial instruction class) were selected from a public elementary school in southern Taiwan to participate in this study. The school is located in a rural area where most of the parents of these students are farmers. There were 17 students (8 boys and 9 girls) in each class. The mean ages for the two classes were 10.14 and 10.17 years, respectively.

Based on a pretest (see below), students in both groups were rank-ordered. The top quintile of each class was considered the high-level group, the students in the middle quintile (40%-60%) formed the middle-level group, and the bottom quintile was the low-level group. In each class, two students from each level were randomly selected and interviewed before and after the remedial instruction. A total of 12 students were interviewed. Each student was required to answer 10 interview questions to confirm their problem-solving methods.

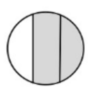
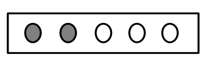
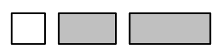
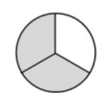
The interviewed students in the NS-based remedial instruction class were labeled as EH1 and EH2 for the high-level group, EM1 and EM2 for the middle-level group, and EL1 and EL2 for the low level group. Students in the textbook-based remedial instruction class were labeled as CH1 and CH2 for the high-level group, CM1 and CM2 for the middle-level group, and CL1 and CL2 for the low-level group.

Instruments

Number sense test. The same number sense test was used as the pretest and the posttest. The test was designed by Yang & Li (2008) and included 25 number sense questions based on the 5 number sense components defined above. Each question included answer options and reason options. More specifically, the reason options included “number sense method”, “rule-based method”, “misconception”, and “guess”. For example, the following question was included on the number sense test:

1. In which answer below does the shaded area best represent 2/3 of the total area?

   Answer options:

(1)  (2)  (3)  (4) 

Reason options:

- Because 2/3 means something was equally divided into 3 pieces and 2 pieces are shaded.
- Because 2/3 means 2 circles were shaded and 3 circles were not shaded.
- Because 2/3 means a cake was divided into 3 pieces with 2 pieces shaded and 1 piece unshaded.
- Because 2/3 means a circle is divided into 3 parts with two parts shaded.
- I am guessing.

The Cronbach’s α coefficient for the test is 0.853. The construct reliability via SEM is 0.805, reflecting good convergent validity of content.

Interview instrument. A pre-interview and a post-interview were conducted, both of which used the same 10 questions (see Appendix 1) selected from the number sense test.

Interview procedure. Several specific questions were used during the interview: 1. Can you justify your answer? 2. Can you tell me your reasons? 3. Can you tell me how to do it? 4. Can you do it in another way? These specific probes were developed to examine each student’s thinking process and identify the specific method used in each number sense solution. A pilot study was carried out with a different class of third graders. The pilot study indicated that these questions worked well at determining which methods were utilized by the students when answering number sense related questions.

During each individual interview, a student was given an interview booklet. Each page included one item with ample space to allow students to write out any ideas if necessary. Specifically, the following directions were given:

1. Think aloud and show me your method when solving the problem.

2. Do not turn to the next page without permission.

Each interview lasted about 40 minutes. In addition, the interviews were videotaped and later transcribed.

Treatment

Of the two classes of 3rd graders in the study, one class was assigned to receive NS-based remedial instruction. The other class received textbook-based remedial instruction. Both class teachers had nine years of teaching experience at the time when the study was conducted. The teacher of the NS-based remedial class took a semester-long course which focused on number sense teaching and learning in order to know how to design and implement NS-based remedial instruction for this study. The teacher of the textbook-based remedial class took a semester-long course which focused on the national curriculum in order to know how to design and implement textbook-based remedial instruction for this study. The teacher for the NS-based remedial class encouraged her students to have small-group discussions and whole class discussions, promoted the students' comprehensive explanations, and encouraged the children to question, debate, and provide arguments to support their solutions in order to help them develop a better understanding of number sense. In contrast, the instruction used in the textbook-based remedial class focused on drills and practice to promote the acquisition of procedural knowledge. Thus, the major difference between the two modes of teaching in this study was the type of instructional focus and teaching materials used.

Activities Conducted in the NS-based Remedial Instruction Class

The design of the remedial instructional activities was based on the misconceptions of number sense found on the pretest. Each item in the pretest included both answer options and reason options. Thus, when a student completed an item, we knew the reason why he or she chose the answer. Hence, we designed the remedial instructional activities based on the information revealed from the students' responses on the pretest. In other words, the number sense test provided a platform for teachers and students to confront specific number sense misconceptions that the students displayed.

Each of the number sense components described above was addressed in three to five activities. There were 20 NS-based remedial instruction activities in total and each activity lasted about one class period (40 minutes per period). Each week there were 3 class periods spent on implementing these activities; the teaching experiment lasted about 7 weeks. One of the activities designed to remedy the misconceptions of students in the NS-based remedial instruction class was related to place values. We describe this activity here:

A number wheel is a concrete teaching tool that is composed of wheels representing the thousands, hundreds, tens and ones places. Each number wheel can be rolled from 0 to 9. This activity helps students recognize 4-digit whole numbers. By operating the number wheel, students are able to discover the greatest and the least value of 4-digit whole numbers (see Figure 1). During the activity, the following questions were discussed:

(1) Find the greatest 4-digit whole number and the lowest 4-digit whole number.
How did you determine your answers? ____

(2) Is "0417" a 4-digit whole number? (   ) Why? ____

(3) Find the greatest 5-digit whole number and the lowest 5-digit whole number.

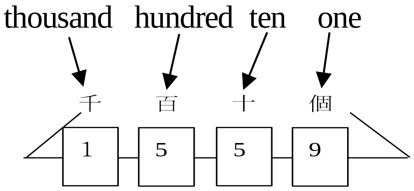


Figure 1. The number wheel used in the NS-based remedial instruction class

Activities Conducted in the Textbook-based Remedial Instruction Class

The textbook-based remedial instruction class also included 20 activities from five units of the 3rd grade textbook. These learning units were developed based on the misconceptions or deficient knowledge found on the pretest. The five units used in the textbook-based remedial instruction class were "any number under ten thousand", "four-digit whole numbers addition and subtraction", "multiplication and division", "perimeter and proportions" and "fractions". One of the examples used in a textbook-based remedial instruction activity is shown in Figure 2:

An Example: The following long dragon is held up by 3760 persons. It looks like the dragon has 7520 feet. Use the following place chart to represent this number.

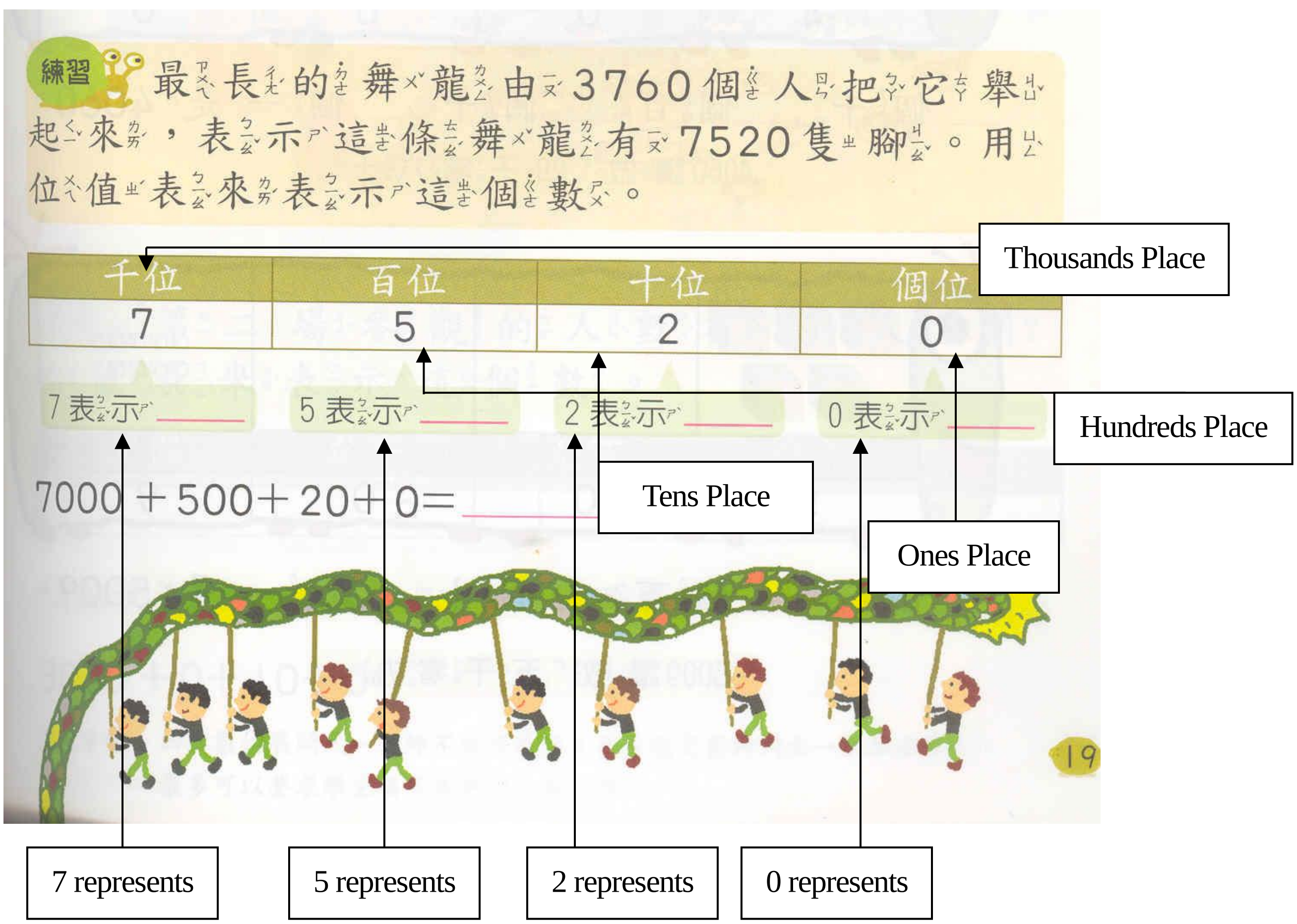


Figure 2. Example of an activity chart utilized for textbook-based remedial instruction.

The teacher in the textbook-based remedial instruction class invited the students to discuss the meanings of 7, 5, 2, and 0 in the number 7,520 and then asked them to do the following exercises:

Please place >, =, or < in the following boxes (1) 5678□8765; (2) 3502□3205; (3) 1987□2001; (4) 9001□9999.

Data Collection and Analysis

Data collected for this study included the pretest, posttest, pre-interview, post-interview, and the teaching experiments. All of the interviews and teaching experiments were video recorded and later transcribed.

Each item in the number sense test was given 4 points for a correct answer and 0 points for an incorrect answer. The test included 25 items so the total score was 100 points. Each student's response was examined and assigned to one of the following categories:

1. Number sense-based — the student utilized one or more of the five components of number sense in their solution method (e.g., using benchmarks appropriately, recognizing number magnitude).
2. Rule-based — the student applied rules of standard written algorithms only.
3. Couldn't explain — the student was unable to give a clear explanation for his/her method of solving a particular problem.
4. Incorrect — the student gave a wrong answer no matter which strategy was used.

The pretest and posttest data were analyzed using ANCOVA and chi squared tests.

The first two authors conducted the pre- and post-interviews. They then analyzed the transcripts and independently coded students' responses into the four categories above. Initial consistency-checking produced an inter-rater categorization agreement of about 90%. The remaining responses were re-examined and discussed by the authors to resolve which of the above criteria best described the participant's responses.

Results

The Learning Module for NS-based Remedial Instruction

A variety of student misconceptions were identified during the pretest and pre-interview. These guided the design of the remedial instruction. For example, the pretest indicated that some students had trouble in determining the greatest and the least 4-digit whole numbers. Many students believed that the greatest 4-digit whole number was 9900, 9000, or 9990 or that the least 4-digit whole number was 1111, 1011, or 1001. In order to correct this type of misconception, a specific learning activity called "the Number Wheel" was designed to reconstruct students' number sense.

The following excerpts from the NS-based remedial instruction show how the teacher helped students gradually develop a better understanding of place value through the use of the Number Wheel. Seven concrete steps were taken which are explained below.

1. *Please show us the greatest and the least 4-digit whole numbers*
   To effectively help children develop their number sense, the activity was based on a process-oriented teaching approach, which required students to defend, query about, and prove their ideas during small-group problem solving as well as whole class discussions.
2. *Posing a challenging problem and encouraging students to ask questions*
   After presenting the question, students were asked if they understood the meaning of the question. By asking questions, students are able to gain additional information before solving a problem. Below is one such example:
   T: Any questions about the problem?
   S: What is the meaning of "find the greatest 4-digit whole number"?

T: When you turn the number wheel, you may find the greatest 4-digit whole number. In addition, you should write down your reasons and show me why you think your answer is correct.

S: So we can find the greatest 4-digit whole number by turning the number wheel, and we should try to write down our thinking process and reasons why we think this way.

T: Yes, you got it. Any other questions? (There are no more questions.) Now let's start small-group discussion and decide your answer and corresponding reasons.

3. *Small-group discussion: to revise and reconfirm the answer through querying*

S1: The greatest 4-digit whole number is 9999 and the least 4-digit whole number is 1111.

S2: The least 4-digit whole number should be 1100. 1100 is smaller than 1111.

S3: No! No! The least 4-digit whole number should be 1000 which is smaller than 1100. And there is no other 4-digit whole number which is less than 1000.

S2: Well! Why didn't I roll 1000 a moment ago? I want to roll it one more time. *(S2 rolls the Number Wheel again.)* The number 1000 really exists! Yeah! 1000 is smaller than 1100! *(S2 shouts after rolling the Number Wheel.)*

Although S2 could not find the correct least 4-digit whole number in the beginning, he was able to find and understand the least 4-digit whole number after discussion. The cooperation of his group members encouraged S2 to roll the Number Wheel again and confirm the least 4-digit whole number. This shows that the small-group discussion was able to effectively promote students' positive thinking through collaboration and communication. As students continued their discussion, the teacher listened patiently:

S1: Are you sure there isn't a smaller 4-digit number than 1000?

S2: You can turn the Number Wheel again. When the thousandth-wheel is rolled to 1, and the rest of the wheels are rolled to 0, it produces the least 4-digit whole number. Any number smaller than 1000 should be without thousands digit, and it would be a 3-digit whole number. So, 1000 is the answer.

S1: Is the greatest 4 digit-number 9999?

S4: Yes! The greatest number of each place number is 9, so we roll each place number of the Number Wheel to 9 to get the greatest 4-digit whole number. (*He looked at the number wheel which was rolled to 9999.*)

To get the correct answer, the group members continuously addressed different views or doubts. With collaboration, the group members mutually benefited from each other by eliminating contradictions and explaining their own answers through discussion. Right after the small group discussion, the teacher guided students to discuss "whether 0417 is a 4-digit whole number or not." Part of the resulting whole-class discussion is given below:

4. *Promoting comprehensive explanations*

S5: We think 0417 is a 4-digit whole number.

T: How do you know 0417 is a 4-digit whole number?

S5: Because 0417 has four digits, it is definitely a 4-digit whole number. (He points to each number of the Number Wheel and counts 1, 2, 3, 4.)

T: No matter whether your answer is correct or not, you should be able to explain how you arrive at that answer. In order to be understood by your classmates, please explain your own reasons clearly.

S5 He didn't explain how he got the answer since he only addressed what the answer was originally. The teacher continued to ask students to explain their reasoning to "Whether 0417 is a 4-digit whole number or not?" and encouraged them to explain step by step.

5. *Encouraging students to question, debate, and prove their solutions*

   T: Are there any comments to the answer of S5?

   S6: S5 said that 0417 was a 4-digit whole number, and it isn't true. The thousandth digit of 0417 is 0, so 0417 should be a 3-digit whole number.

   S5: Why can't the thousandth digit be 0?

   S6: Of course it can't be! Although there is a digit of 0 in the thousandth place, it means that the real number is just 417. It's a 3-digit number. (He points to the number wheel and explains at the same time.)

   S5: But the number is 0417 which has four digits. The number is different from 417!

   (S5 and S6 argued about this issue nonstop. The teacher skillfully asked S5 to connect the number 0417 to a real-life situation, and required S5 to rethink his answer.).

   T: S5, could you try to think of an example of 0417 found during our daily life?

   S5: Yes! My father picks me up to withdraw money from an ATM machine, and my pin number is 0825. If I put in 825, I can't withdraw money. So 0825 is different from 825, and my father urges me to remember that the code is a 4-digit whole number.

   The teacher intentionally asked a question in an attempt to invoke S5's cognitive conflict so that S5 could recognize the difference between a 3-digit and 4-digit whole number. However, S5 brought up a pin number as an example of 0825. It seems difficult for S5 to understand why 825 is the same as 0825.

   T: S5 mentions a special case. Let's think about the difference between a "4-digit whole number" and "4 digit pin numbers". When we set up a pin number, like for a safe deposit box or a mailbox, etc., we can put "0" in the forefront. S5 says his ATM card pin number for withdrawing money is 'zero-eight-two-five', which is not '0 thousand eight hundred and twenty-five' (Some students expressed that "0-thousand" should be abbreviated to "eight-hundred twenty-five"). This means that the forefront digit of a 4 digit pin number can be "0"; however, can the forefront of a 4-digit whole number be "0"? (Students said "no way".)

S5 showed a misconception of the structure of a 4-digit whole number. The teacher tried to further interpret this example by using an additional example of a combination lock or pin number used for safe deposit boxes or mailboxes.

6. *Encouraging students to develop different strategies and draw a conclusion*

   T: Could you pose a different explanation for a 4-digit whole number?

   S7: After we rolled the Number Wheel, we found the greatest 4-digit whole number was 9999 and the lowest 4-digit whole number was 1000. So could I say that all numbers between 1000 and 9999 are 4-digit whole numbers?

   T: Of course! In the range of 4-digit whole numbers, 1000 is the least and 9999 is the greatest. S7 did a good job. Are there any other definitions?

   S2: A 4-digit whole number is composed of ones, tens, hundreds and thousands, and each number must be from 0 to 9.

   S1: Any number which exceeds 1000 and doesn't exceed 9999 is a 4-digit whole number.

The teacher continuously guided students to generate different explanations for 4-digit whole numbers. Students enthusiastically expressed their thoughts. It is worth noting that the teacher was patient and gave plenty of time for students to elaborate on their answers. This gave the teacher a chance to regulate and direct students' thinking in a reasonable manner.

7. *Posing a more difficult problem to confirm students' understanding*

   T: Please think about "what are the greatest and the least 5-digit whole numbers?"

   S3: The greatest 5-digit whole number is 99999, and the least 5-digit whole number is 10000.

T: Good! The greatest 5-digit whole number is 99999, and the least 5-digit whole number is 10000. Raise your hand if you got the same answer. (Only 2-3 students did not raise their hands.)

The teacher posed a more difficult question to confirm students' understanding of place value. Most of them were able to answer correctly, and it was evident that these students made progress after the NS-based remedial instruction.

Teaching reflections. The teacher in the experimental class said: "Although the manipulations and the Number Wheel helped students to think about the whole number digits, it also confused students when they needed to judge digits that occurred in special situations." For example, in the case of S5, the number wheel couldn't help him to differentiate "825" and "0825". It would be a good idea to change the color of "0" in the thousands place of the number wheel to highlight the difference between "825" and "0825". It is helpful to identify the difference between 4- and 3-digit whole numbers. Additionally, the activity only guides students to discuss the digits "0417" in order to help students recognize that the first digit of a whole number cannot be "0". Maybe students should have the opportunity to discuss more questions that lead them to consider the difference between "0417" and "4017". This could help students gain a clearer understanding of 3- and 4-digit whole numbers. Finally, when giving students instructions, we should put more emphasis on reading aloud whole numbers for students to mentally capture the concept of whole numbers.

Differences on Number Sense Performance

Table 1 summarizes the descriptive statistics for the pre- and posttests for both classes. The pre-test mean scores for the NS-based remedial instruction class and the textbook-based remedial instruction class on the pretest were 57.41 and 59.29 (out of 100 points), respectively. These scores indicate that the students' performance on number sense was relatively poor and they imply that in-time remedial instruction was clearly needed for these students. After the remedial instruction, the mean scores on the number sense test for the NS-based remedial instruction class and textbook-based remedial instruction class increased by 20.94 and 3.77 points, respectively. Thus, the students in the NS-based remedial instruction class made greater progress on number sense than students in the textbook-based remedial instruction class.

Table 1
Descriptive Statistics for the NS-based Remedial Instruction Class and Textbook-based Remedial Instruction Class

Group	N	Pretest		Posttest	
		M	SD	M	SD
NS-based	17	57.41	19.39	78.35	13.79
Textbook-based	17	59.29	17.96	63.06	15.72

Note.
NS-based: NS-based remedial instruction class
Textbook-based: Textbook-based remedial instruction class

Table 2 reports the results of ANCOVA for NS-based remedial instruction class and textbook-based remedial instruction class. The pretest score was the covariate in this analysis. There was a statistically significant difference (F = 107.388, p=.000 < α=.05) between the posttest mean score of students who received NS-based remedial instruction and that of students in the textbook-based remedial instruction class. This implies that the NS-based remedial instruction had a significant positive impact on students' learning of number sense.

Table 2
Results of One-Way ANCOVA

Sources of variation	*df*	Mean Squares	*F*	*P*
Covariance (pretest)	1	6213.383	246.487	.000
Group	1	2363.568	93.763	.000
Error	31	25.208		

Note. $R^2 = .916$

Differences in the Use of Number Sense Methods

Table 3 reports the pre- and post-interview results of students' responses from both classes to the 10 interview questions. Students in the NS-based remedial instruction class used number sense more frequently to answer interview questions after remedial instruction than before. For example, EH1 (from the high-level NS-based group) was able to apply number sense-based methods to only half of the questions for the pre-interview; however, EH1 and EH2 applied number sense-based methods to answer all questions on the post-interview, respectively. In contrast, CH1 (from the high-level textbook-based group) only had a few changes on the use of NS-based methods and CH2 showed no change in the use of number sense-based methods after remedial instruction.

Table 3
Frequencies of Methods Used by Students for Both Classes on the Pre- and Post-interviews

	NS-based remedial instruction class						*Textbook-based remedial instruction class*					
Methods	EH1	EH2	EM1	EM2	EL1	EL2	CH1	CH2	CM1	CM2	CL1	CL1
Pre-interview												
Correct												
NS-based	5	3	1	0	0	0	6	3	1	1	0	0
Rule-based	5	4	1	3	1	1	4	4	2	3	1	1
Couldn't explain	0	0	1	0	0	0	0	0	1	0	0	0
Incorrect	0	3	7	7	9	9	0	3	6	6	9	9
Post-interview												
Correct												
NS-based	10	10	9	9	6	4	7	3	2	2	0	0
Rule-based	0	0	0	0	0	0	3	5	2	3	2	2
Couldn't explain	0	0	0	0	0	0	0	0	0	0	0	0
Incorrect	0	0	1	1	4	6	0	2	6	5	8	8

Note.

1. The same10 questions were asked for both interviews.
2. EH1 & EH2 represent students in the high-level group on the number sense pretest, EM1 & EM2 represent those in the middle-level group on the number sense pretest, and EL1 & EL2 represent those in the low-level group on the number sense pretest in the NS-based remedial instruction class.
3. "Couldn't explain" indicates that students couldn't give a clear explanation on how they solved a problem.

Moreover, it was difficult for EM1 and EM2 (the middle-level group) in the NS-based remedial instruction class to adopt number sense-based methods to answer questions during the pre-interview, but on the post-interview they were able to apply number sense methods to answer nine out of ten interview questions. However, CM1 and CM2 (the middle-level group) in the textbook-based remedial instruction class only utilized number sense-based methods to solve two problems on the post-interview. Similarly, the low-level students in the NS-based remedial instruction class also had greater changes in their use of number-sense methods than the low-level students in the textbook-based remedial instruction class after the remedial instruction. These interview results indicate that students at every level in the NS-based remedial instruction class made progress on the use of number sense after the NS-based remedial instruction. In contrast, students at all three levels in the textbook-based remedial instruction class made little progress on the use of number sense after remedial instruction.

EH1's remarkable changes between the pre- and post-interview for question 5 illustrate the developments in the NS-based class:

Q5. Without using paper and pencil, which answer is the best choice to the problem "120×4＋120"?

(1) 120×124　　(2) 120×5　　(3) 12×4+10×4＋12×10　　(4) 240×4

In both the pre-interview and the post-interview, after EH1 completed the test item, the researchers asked EH1 to explain his answer.

Pre-interview

T: Why did you think "120×4＋120" was equal to "120×5"?

EH1: Because 120 multiplied by 4, plus 120 equals 600.

T: How did you get your answer for this test item?

EH1: I used paper and pencil to find the exact answer.

T: Are you able to use a different way to find the answer?

EH1: I am not sure.

Post-interview

T: Why did you think "120×4＋120" equaled "120×5"?

EH1: "120×4" means there are 4 "120", then plus "120", so the total is 5 "120". It is the same as "120×5".

During the pre-interview, EH1 could only use paper-and-pencil to get the answer. However, during the post-interview, EH1 showed a conceptual understanding of "120×4＋120." In short, EH1 showed a better understanding of numbers, operations, and their relationships.

Similarly, the following interview excerpts from EM1's responses about question 6 showed a better understanding of numbers, operations, and their relationships after the NS-based remedial instruction.

Q6. In celebration of Toytown's anniversary, all toys are on sale. One mother bought a model car that cost $199, and a doll that cost $399. Without using paper and pencil, how many one-hundred dollar bills are needed to pay for the mother's purchase?

(1) 4 (2) 5 (3) 6 (4) 7

Pre-interview

T: Why did you choose 5?

EM1: Because “199+399” is equivalent to “598” (Looking at the rule-based method used by himself)

T: Could you clearly explain why you chose 5 hundred-dollar-bills?

EM1: “598” includes 5 one-hundred-dollar bills!

Post-interview

T: Why did you think that 6 hundred-dollar-bills were needed?

EM1: Because “199” is about two hundred and “399” is about four hundred. Therefore, the mother needs 6 hundred-dollar-bills to pay for them.

Students such as EL1 and EL2 at the lower level of ability also made progress in number sense, as shown in these responses of EL1 about question 9:

Q9. What is the difference between “3 × 140” and “4 × 140”? (1)1 (2)140 (3) 40 (4)10

Pre-interview

T: Why did you choose “140”?

EL1: “3 times 140” is equal to “140 multiplied by 3”; “4 times 140” is equal to “140 multiplied by 4”, and then I subtract them from each other to get the answer.

T: How did you arrive at your answer?

EL1: I used a rule-based method to find the exact answer. Because 140x3=420 and 140x4=560, 560–420=140. Therefore, the answer is 140.

Post-interview

T: Why did you answer “140”?

EL1: Because “3 times 140” means that there are three “140”, and “4 times 140” means that there are four “140”, the difference is one “140”.

Compared with EL1’s performance during the pre-interview, EL1 has clearly learned by the post-interview how to flexibly apply number sense to solve questions.

Table 4 summarizes the frequencies of different problem-solving methods used by the students in the interviews in both the NS-based remedial instruction class and the textbook-based remedial instruction class. During the pre-interview, students in the NS-based remedial instruction class used number sense-based methods 9 times (15%), rule-based methods 15 times (25%), could not explain once (1.7%), and gave incorrect answers 35 times (58.3%). During the post-interview after the NS-based remedial instruction, these students used number sense-based methods 48 times (80%) and provided 12 incorrect answers (20%). Clearly, most methods switched to number sense-based methods for these students. This was a significant change in the use of problem solving methods (χ^2=39, df=6, p=.000 < α= .05, using Bowker’s test of symmetry).

During the pre-interview, the students in the textbook-based remedial instruction class used number sense-based methods 11 times (18.3%), rule-based methods 15 times (25%), could not explain once (1.7%), and gave 33 incorrect answers (48.3%). During the post-interview after the textbook-based remedial instruction, these students used number sense-based methods 14 times (23.3%), rule-based methods 17 times (28.3%), and gave 29 incorrect answers (48.4%). Bowker’s test of symmetry (χ^2=6, df=6, p=.423 > α= .05) showed that there was no significant change in the use of different problem-solving methods between the pre- and post- interviews.

Table 4

Summary of Frequencies of Different Methods Used in the Pre- and Post-interviews for Both Classes

	NS-based remedial instruction class		Textbook-based remedial instruction class	
	Pre-interview	Post-interview	Pre-interview	Post-interview
Correct				
NS-Based	9 (15%)	48 (80%)	11 (18.3%)	14 (23.33%)
Rule-Based	15 (25%)	0 (0%)	15 (25%)	17 (28.3%)
Couldn't explain	1 (1.7%)	0 (0%)	1 (1.7%)	0 (0%)
Incorrect				
NS-Based	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Rule-Based	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Couldn't explain	35 (58.3%)	12 (20%)	33 (48.3%)	29 (48.4%)

Note: The same 12 students were interviewed for both the pre- and post-interviews. Each student was required to answer 10 questions.

In summary, the results of interviews showed that after NS-based remedial instruction, the students made progress in the application of number sense. The NS-based remedial instruction was able to effectively increase the use of number sense while problem solving. In contrast, students in the textbook-based remedial instruction class showed few changes in their use of number sense after instruction.

Discussion and Conclusion

The major finding of this study is that, compared with students in the textbook-based class, students in the NS-based remedial instruction class made significant progress on number sense performance after the NS-based remedial instruction. Moreover, the interview results show that the students in the NS-based remedial instruction class were able to apply number sense more frequently and efficiently than students in the textbook-based remedial instruction class. These findings imply that students' number sense can be promoted through effective teaching with appropriate teaching materials. The type of remedial instruction described in this paper illustrates an approach which supplied third graders with an opportunity to remedy their deficiencies with number sense. This confirms the results of several earlier studies (Floden, 2002; Hiebert & Grouws, 2007; Yang & Li, 2008; Yang & Wu, 2010) indicating that giving children an opportunity to learn is the most important predictor of number sense performance.

In particular, the participants' misconceptions were corrected through a learning environment that encouraged the discussion and elaboration of answers. This demonstrates that students' number sense can be improved through meaningful learning. This further supports the findings of earlier studies (Yang, 2006; Yang & Wu, 2010) that effective teaching strategies can promote students' number sense. Moreover, this result highlights the critical role of the teacher as a facilitator in the NS-based remedial instruction class, posing challenging questions, eliciting each student's thinking processes, listening carefully to students' ideas, and asking students to clarify and justify their ideas (NCTM, 2000; Yang, 2006). In addition, the Number Wheel teaching episode shows that students can develop a better understanding of number sense if they have opportunities to use manipulatives. This approach is quite different from that of the textbook-based remedial instruction class in which students did not have an opportunity to use manipulatives.

In sum, effective remedial instruction on number sense for third graders in Taiwan is possible, and it can be done through a series of practical teaching episodes. The teaching episodes reported here show that practical remedial instruction is helpful in promoting children's number sense. Indeed, this can be a good model for teachers to learn how to effectively guide their students in self-reflection and discussion in future instruction. We believe that this NS-based remedial instruction can not only have a profound impact on number sense for low achievers, but also radically replace the mundane drill and practice type of traditional textbook-based remedial instruction.

The educational implications of this study are threefold. First, based on students' misconceptions or deficient knowledge (e.g., the misconception of place values displayed by students on the pretest), it is clear that well-designed remedial learning materials should be offered just-in-time. Currently, few remedial instruction models for third graders are available. The remedial instruction presented in this study will encourage teachers to conduct number sense-based remedial instruction. Second, during remedial instruction, allowing students to explore and manipulate teaching aids such as the Number Wheel appropriately will support students' engagement when solving problems. Third, "children's poor performance on number sense not only exists at middle grade levels, but also emerges at lower grade levels in elementary school" (Yang & Li, 2008, p. 453). This study demonstrates the feasibility and importance of implementing remedial instruction in number sense at an earlier age.

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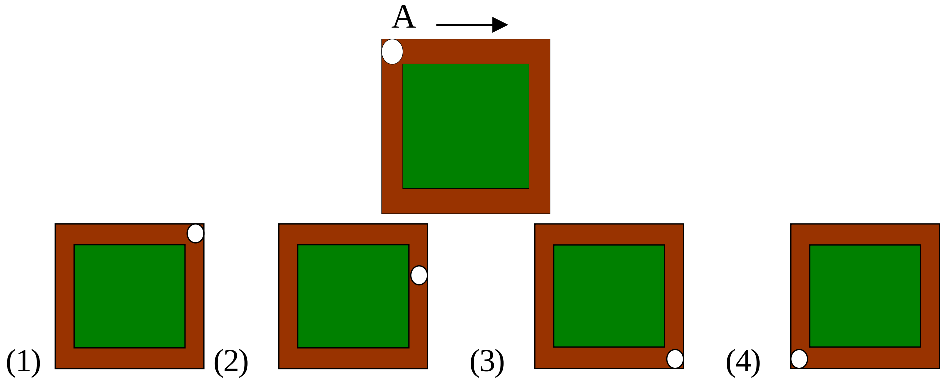
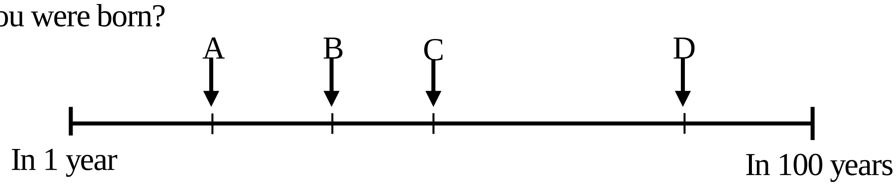
Appendix 1

The 10 interview questions:

1. Imagine a park that is square like the picture below. Suppose you start to walk from point A along the direction of the arrow, where would you be after going one-third of the way around the park?



2. Which letter (A, B, C, D) in the number line could best represent (or be closest to) how many years ago you were born?



(1) A (2) B (3) C (4) D

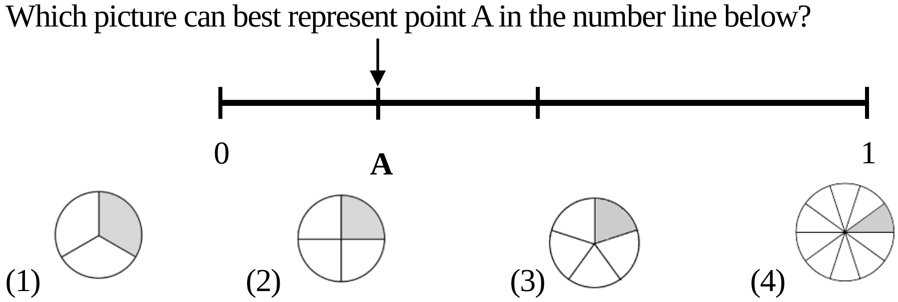
3. Which of the following is equal to eight tenths?
(1) two four-fifths (2) eight one-tenths (3) ten one-eighths (4) eight and one-tenth

4. An older brother's bank account balance shows the smallest possible four-digit number, and his younger brother's balance shows the biggest possible three-digit number. What's the difference of their account balances?

5. Without using paper and pencil, which answer is the best choice to the problem "120×4＋120"？
(1) 120×124 (2) 120×5 (3) 12×4+10×4＋12×10 (4) 240×4

6. In celebration of Toytown's anniversary, all toys are on sale. One mother bought a model car that cost $199, and a doll that cost $399. Without using paper and pencil, how many one-hundred dollar bills are needed to pay for the mother's purchase?
(1) 4 (2) 5 (3) 6 (4) 7

7. Which picture can best represent point A in the number line below?



8. If 45 apples can be packed into one box and a customer wants to buy 8 boxes of apples, but later he requests all apples should be packaged into 45 boxes. How many apples should be packaged into one box?
   (1) 5 (2) 53 (3) 8 (4) 45

9. What is the difference between "3 times 140" and "4 times 140"?
   (1) 1 (2) 140 (3) 40 (4) 10

10. Which of the following operations results in the largest product?
    (1) 321×4 (2) 320+321+322+323 (3) 900+300 (4) all are equal