Errors in Solving Word Problems about Speed: A Case in Singapore and Mainland China

Chunlian Jiang
University of Macau, China

Abstract

Error patterns often reveal the underlying misunderstanding of mathematics concepts, lack of problem-solving strategies, and/or immature problem-solving strategies. In this study, Fong's schematic model was used to analyze errors made by 1,002 Singapore and 1,070 Chinese students in grades 6-8 in solving 11 word problems about speed. We found that 16-92% of the students from both countries could not get the correct answers to the problems. They frequently made E3 (incomplete schema with errors), E4 (using irrelevant procedures), and E5 (no solution) types of errors. The common second level error analysis revealed several similarities between the Chinese and the Singapore students in making errors in computations, misunderstanding of the problems, mismatching of distances and speeds in finding times, and misconception of average speed. The study provides useful information for the teaching and learning of word problems at the elementary level.

Keywords: speed, misconceptions, error analysis, word problems, comparative studies

The study of pupils' problem-solving errors in different topics is a prominent field in mathematics education (Ashlock, 1998; Babbitt, 1990; Booth, 1983; Cox, 1975; Engelhardt, 1977; Olivier, 1989; Radatz, 1979, 1980; Roberts, 1968). Errors lead to wrong answers. They are systematic when that they are applied regularly in the same circumstances (Olivier, 1989), and they can be resistant to casual re-education (Booth, 1983). Such errors are caused by underlying conceptual structures which are called misconceptions (Olivier, 1989). Students' errors are not simply a result of ignorance or carelessness. They are often caused by an overgeneralization of previous knowledge that is correct in an earlier domain to an extended domain that is not valid (Olivier, 1989; Radatz, 1980). Diagnostic error analysis not only provides information about individuals' mathematics learning, but also provides practical help for teachers with regard to individualized instruction. These kinds of analysis remind teachers to be sensitive to the effects of individuals' previous learning and to make an effort to connect new knowledge to previous learning (Olivier, 1989; Radatz, 1979). This study investigates errors students make when solving word problems about speed at the elementary level.

The topic of word problems about speed was selected for study because these problems apply various mathematical concepts from the primary to the university level (Bowers & Nickerson, 2000; Ministry of Education (MOE) (Singapore), 2000a, 2000b; Nichols, 1996; People's Education Press (PEP), 1994; Teh & Looi, 2002a, 2002b; Tylee, 1997). Several studies have included rate problems as a specific model of multiplication and division (Bell, Fischbein, & Greer, 1984; Fischbein, Deri, Nello, & Marino, 1985; Greer, 1992). However, the word problems about speed included in these studies were from only the simplest category of the 13 categories of motion (speed) problems that Mayer (1981) identified. Mayer analyzed algebraic word problems including those about speed in secondary school mathematics textbooks, but Mayer did not investigate how students actually solve the problems and what difficulties they may have. This study seeks in part to fill these gaps.

Correspondence should be sent to Chunlian Jiang, University of Macau, China. Email: cljiang@umac.mo

The participants in this study came from grades 6-8 because similar word problems about speed are presented in the mathematics textbooks for these three grades (L. Jiang, 1998a, 1998b; PEP, 1992, 1993a, 1993b; MOE (Singapore), 2000a, 2000b; Teh & Looi, 2002a, 2002b).

This study also endeavored to reveal the similarities and differences between the students in the two different contexts (Singapore and China). Cross-national studies provide us with an opportunity to ascertain the strengths and weaknesses of educational systems (Robitaille & Travers, 1992), and consequently provide information about how to improve the teaching and learning of mathematics (Cai, 2000a, 2004; Robitaille & Travers, 1992). Children from Singapore and China have performed exceptionally well in international comparative studies in mathematics (Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzalez, E. J., Kelly, D. L., & Smith, T. A., 1997; Zhang, 1998). Singapore was ranked first to third in mathematics among the participating countries for grades 4 and 8 in the Trends in International Mathematics and Science Studies (TIMSS) in 1995, 1999, 2003, and 2007 (Beaton, A., Mullis, I., Martin, M., Gonzalez, E., Kelly, D. & Smith, T., 1996; Mullis et al., 1997; Mullis, Martin, & Foy, 2008; Mullis, Martin, Gonzalez, & Chrostowski, 2004; Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., et al., 2000). In the Second International Assessment of Education Progress (SIAEP), China was ranked first among the twenty-one participating countries for 13-year-olds (Zhang, 1998). The Programme for International Student Assessment (PISA) found that Chinese students from Shanghai performed the best (OECD, 2010). A cross-national study between these two top-performing countries can provide us with useful information on the strengths and weaknesses of their systems of mathematics education. This will broaden our experience and provide different perspectives for addressing practical issues related to the teaching and learning of mathematics (Cai, Lin, & Fan, 2004).

To summarize, this study attempts to investigate the errors made by Chinese and Singapore students in grades 6 to 8 when solving word problems about speed. Specifically, we want to address the following research question: What types of errors do Chinese and Singapore students in grades 6 to 8 make when solving word problems about speed?

Fong's Schematic Model for Error Analysis

Error analysis has a long history in mathematics education. Error patterns often reveal underlying misunderstandings of mathematical concepts, lack of problem-solving strategies, and/or immature problem-solving strategies (Babbitt, 1990). Radatz (1980) made an extensive list of more than 80 studies in this area from the beginning of the twentieth century to the end of the 1970s. Radatz concluded that arithmetic constituted the dominant subject matter for the majority of error analysis studies. Clements (1980) discussed several classifications of errors made in solving word problems from the 1920s to the 1970s including those of Newman and Casey. Newman (1977) and Casey (1978) classified students' errors in solving word problems in terms of reading, comprehending, transforming, processing, and encoding. These error analyses in both computational tasks and word problems are important to mathematics education because they provide useful information for effective teaching and learning. They not only indicate what goes wrong, but also suggest to us that what we do may lead to students' errors. In addition, they suggest ways we can help students eradicate their misconceptions.

Fong's (1995) schematic model for error analysis is based on the schematic approach for analyzing students' strategies in solving both computational and word problems. Fong defined a schema as the network of interrelationships between different sets of knowledge that constitute a concept and schemata as data structures that represent the generic concepts stored in memory.

Fong classifies errors into two levels. The first level is categorized in terms of schematic approach into five categories: (E1) complete schema with errors, (E2) incomplete schema with no errors, (E3) incomplete schema with errors, (E4) using irrelevant procedures, and (E5) no solution.

The second level of errors is categorized into four categories: (a) language, including reading and comprehension, (b) operational, including encoding and transformation, (c) mathematical themes such as basic facts, algorithms, and concepts, and (d) psychological factors including motivation and carelessness. Psychological factors are always important factors that affect students' problem-solving activities. However, they are difficult to identify from students' written solutions. Therefore, this study focused on the comprehension, operational, and mathematical themes.

The error categories were synthesized from the literature on error analyses in computational and word problems. Fong's (1995) model emphasizes the importance of schematic knowledge to mathematical problem solving. He pointed out the second level of errors could be subsumed under the E1, E3, and E4 categories. He further argued that a pupil must first overcome the first level of errors in order to successfully solve problems.

The characteristics of the five first-level categories are briefly described below using one problem from the current study as an example. The four incorrect solutions, representing categories E1, E2, E3, and E4, are presented below together with one correct solution. By comparing these incorrect solutions with the correct one, we can see what goes wrong in the incorrect solutions. All of these solutions used arithmetic strategies.

The Problem:	Mike made a journey from City P to City Q. In the first hour, he covered $\frac{1}{3}$ of the whole journey. In the second hour, he covered $\frac{1}{5}$ of the whole journey. Finally he took 2 hours to finish the remaining journey at a speed of 42 km/h. Calculate his average speed for the whole journey.
Correct Solution:	$1-\frac{1}{3}-\frac{1}{5}=\frac{7}{15}$, $42\times 2 = 84$ km, $84 \div \frac{7}{15} = 180$ km, $1+1+2 = 4$ hours. $180 \div 4 = 45$ km/h.

In this solution, the following five skills are involved: (a) applying fraction and part-whole concepts to find the fraction of the distance of the third part (D_3) to the total distance (TD), i.e., $\frac{D_3}{TD}$ ($\frac{D_3}{TD} = 1 - \frac{D_1}{TD} - \frac{D_2}{TD}$), (b) using the concept of speed to find D_3, (c) applying the concept of fraction to find TD ($TD = D_3 \div \frac{D_3}{TD}$), (d) applying the part-whole concept to find the total time TT ($TT = T_1 + T_2 + T_3$), and (e) using the concept of average speed to find the answer. Therefore, the solution is coded as a-b-c-d-e, which can be called a strategic path. In comparison, some error categories are described below.

(E1) Complete Schema with Errors

This type of error arises when an error is made in computation or encoding of information although the problem solver is able to connect the relevant schema to the problem's requirement.

Solution 1: $1-\frac{1}{3}-\frac{1}{5}=\frac{7}{15}$, $42\times2 = 84$ km, $84 \div \frac{7}{15} = 190$ km, $1+1+2 = 4$ hours, $190\div4 =$ 47.5 km/h.

Data Entry: a-b-(c)-d-e where letter c with parentheses is the step which contains mistakes.

In this solution, the second level error was operational, i.e., in computation.

(E2) Incomplete Schema with No Errors

In this type of error, students present some, but not all, of the correct steps in the solution. No actual error is made other than incomplete retrieval of a schema leading to a solution. The problem solver has a limited or insufficient schema or is unable to connect all the relevant information that leads to the solution.

Solution 2: $1-\frac{1}{3}-\frac{1}{5}=\frac{7}{15}$, $42\times2 = 84$ km, $84 \div \frac{7}{15} =180$ km.

Data Entry: a-b-c, the student is not able to connect all the relevant information that leads to the solution.

In this solution, there were no second level errors.

(E3) Incomplete Schema with Errors

This category of errors differs from the above categories in that the student makes errors such as computation and/or encoding errors in addition to demonstrating an incomplete schema or an inability to connect all relevant schemata.

Solution 3: $1-\frac{1}{3}-\frac{1}{5}=\frac{7}{15}$, $42\times2 = 84$ km, $84 \div \frac{7}{15} =180$ km,
$180\times\frac{1}{3} = 60$ km, $180\times\frac{1}{5} = 36$ km, $60\div1 = 60$ km/h, $36\div1 = 36$ km/h,
$(60+36+42)\div3 = 46$ km/h.

Data Entry: a-b-c-f-g-h1-h2-(i) where letter i with parentheses is the step which contains mistakes.
The skills f to i are:
f: applying the concept of fraction to find D_1 ($D_1 = TD \times \frac{D_1}{TD}$);
g: applying the concept of fraction to find D_2 ($D_2 = TD \times \frac{D_2}{TD}$);
h1: using the concept of speed to find S_1;
h2: using the concept of speed to find $S_{2;}$
i: using $AS = (S_1 + S_2 + S_3)/3$ to find the average speed for the whole journey.

In this solution, the second level error was made in the mathematical theme category because the student applied a misconception of average speed as the average of individual speeds of the three-motion journey.

(E4) Using Irrelevant Procedures

In this category, the student is unable to retrieve any relevant knowledge or information and apply it to work out the solution. Any knowledge or information that is retrieved has no connection or link to the question, although the problem solver may assume that those pieces of information retrieved are the best solutions without realizing that the connection is erroneous.

Solution 4: $42\times\frac{1}{3} = 14$, $42\times\frac{1}{5} = 8.4$

In this solution, the second level error is in the comprehension and mathematical theme categories because the student takes the speed of the third part as the total distance, which is erroneous.

(E5) No Solution

To Fong (1995), this category refers to a solution which has no written response. In the current study, this category includes both blank responses and solutions where only pieces of information taken from the question are written down without any further work, such as “1 hour = 1/3 of the trip.” In terms of schematic explanation, the student is unable to connect or relate his available schema to the information obtained from the question.

The current study was intended to determine the differences in types of errors made between the students from the two countries, and some common second level errors in solving word problems about speed. This kind of analysis can provide useful information for school teachers to identify areas of weakness of students who fail to solve a problem and to conduct individual remediation accordingly.

Method

Subjects

A total of 1,070 Chinese students (361 in grade 6; 354 in grade 7; and 355 in grade 8) and 1,002 Singapore students (345 in grade 6; 315 in grade 7; and 342 in grade 8) participated in the study. The Chinese sample was from Wuhan City, China. Wuhan is located at the center of China. It is the capital city of Hubei Province. Its development is at the average level of main cities in China. It was chosen as a research site for the researcher’s convenience. The Chinese sample was from three primary and seven secondary schools. The schools were recommended by an officer who worked in the Department of Education of Hubei Province. The classes involved were at the average level of that grade in that school. The Singapore sample was from four primary schools and six secondary schools. The schools were recommended by an officer in the Ministry of Education in Singapore who is very familiar with the Singapore school system. The samples from the two countries were not atypical for their respective regions.

Problems and Administration

A test with 14 items was developed from an analysis of various types of word problems about speed (C. Jiang, 2005). Before the test in this study was administered to the participants, the researcher asked mathematics educators from the Ministry of Education in Singapore as well as Chinese and Singapore school teachers to check whether the students could understand the problems well. Any wording problems that might cause difficulties for students were removed. The test was administered to intact classes. No calculators were allowed. Prior to the test, all the students had learned about and completed the topic of speed.

Data Coding and Inter-coder Reliability

The first three items in the study were short answer questions which did not allow us to use Fong's schematic model for error analysis. Therefore, we only analyzed the other eleven items on the test. The researcher coded all the students' responses in two steps. The first step was to code the responses as correct or one of the five error types; the second step was to code the strategic paths for all the correct and incorrect responses in the manner shown in the examples above. A stratified random sample of test papers was selected from the six samples (country × grade) with 10 from each sample and was coded by another rater, an experienced Singapore teacher, to establish inter-rater reliability. The percentage agreement in the identification of the first level errors was found to be 93%. Discrepancies were resolved through discussion. Coding the strategic paths was tedious, and thus the second rater was not asked to do that.

Although the students in this study came from different grade levels, analysis revealed that the errors they made in solving the problems in the current study were quite similar. Therefore, we do not take grade level as a factor in the following discussion of results.

Results

The results are presented in two parts. First, we present the first level errors made by students from the two countries when solving the eleven problems where showing work was required. Then, we address the common second level errors made by students when solving four of the problems. As all the problems were multi-step problems, students could make errors in almost every step. The errors were too varied to all be covered in depth in this report. Therefore, for convenience of discussion, we chose to report the most common second level errors. These were errors that were made by at least ten students from both countries. Errors that were made by fewer than ten students may not have much practical meaning for effective teaching and learning. By limiting the discussion to the more common errors, we intend to conduct a deeper analysis that will allow us to understand the children's underlying misconceptions.

Differences in First level Errors of Chinese and Singapore Students

Table 1 shows the percentages of Chinese and Singapore students who incorrectly solved each of the eleven problems. The results show that the students had a lot of difficulty solving the multi-step problems in this study. For example about 60% of the Chinese students and more than 90% of the Singapore students could not provide correct answers to Problem 11. It was also found that a higher percentage of Singapore students made errors in every problem except Problem 4, where a similar percentage of students from both countries made errors. A more detailed analysis based on Fong's schematic model will help us to better understand what difficulties the students had in the problem-solving processes and potentially to find ways to help them overcome their obstacles.

Table 1
Percentage of Chinese and Singapore Students who Incorrectly Solved the Problems

Problems		China (n=1070)	Singapore (n=1002)	χ^2 value[1,2]	*p*
1	Incorrect	20	24	4.96	<0.05
2	Incorrect	29	39	24.25	<0.001
3	Incorrect	31	56	129.42	<0.001
4	Incorrect	52	50	0.98	0.322
5	Incorrect	50	63	37.40	<0.001
6	Incorrect	26	49	120.60	<0.001
7	Incorrect	16	74	705.95	<0.001
8	Incorrect	23	71	469.43	<0.001
9	Incorrect	72	86	60.43	<0.001
10	Incorrect	56	80	139.81	<0.001
11	Incorrect	59	92	285.31	<0.001

Note: [1]The chi-square values were calculated from 2×2 contingency tables of the original data with two countries (China/Singapore) and two categories (correct/incorrect). In this table, we only include the number of students who incorrectly answered the problem to avoid replication.

[2]df=1.

Table 2
Total Number and Percentage of Different Types of Errors the Chinese and the Singapore Students Made in Solving the 11 Problems

Error type	China		Singapore	
	No.	%	No.	%
E1	407	9	431	6
E2	250	5	438	6
E3	1770	38	2324	31
E4	1053	23	2730	36
E5	1155	25	1609	21

Note. E1 = Complete schema with errors; E2 = Incomplete schema with no errors; E3 = Incomplete schema with errors; E4 = Using irrelevant procedures; E5 = No solution.

Table 2 shows the total number and percentage of Chinese and Singapore students who made each type of first level error. The total number of errors Singapore students made was more than that made by the Chinese students, although there were fewer Singapore students than

Chinese students in the study. Compared to the Singapore students, higher percentages of the errors made by the Chinese students were E1 ($z=6.47$, $p<0.001$), E3 ($z=8.31$, $p<0.001$), and E5 ($z=4.54$, $p<0.001$). In contrast, a higher percentage of the errors made by the Singapore students were E4 ($z=15.64$, $p<0.001$). There was no significant difference in the percentages of E2 errors made by the students from the two countries ($z=0.98$, $p=0.33$). Overall, the Singapore students were more likely to write down something rather than leave the question unanswered. This is consistent with the behavior of Singapore students in TIMSS 1995 (Kaur & Pereira-Mendoza, 1999) and in the studies of Kaur (1995) and Yuen (1995). In contrast, the Chinese students were less likely to take a risk, which is consistent with other studies (Cai, Lin, & Fan, 2004).

Similarities in Second Level Errors of Chinese and Singapore Students

The eleven problems analyzed in this study can be classified into four groups. We examined students' errors on a representative problem from each group (see Appendix A). Note that three of these are actually algebraic word problems.

The first group (Problems 1 and 6) describes two motions of one object where the directions of the two motions can be assumed to be the same. Problem 1 can be solved using arithmetic strategies, whereas Problem 6 cannot. It is a typical algebraic word problem like the Cows and Chickens Problem (Kaur, 1998). As we can see from Table 1, more students made errors in Problem 6 than Problem 1. Therefore, we chose Problem 6 as one example for the two problems.

The second group (Problems 5, 10, and 11) describes a round trip, where one object made two motions with the same distance but different directions. Problem 5 can be solved using arithmetic strategies. However, Problems 10 and 11 cannot. In Problem 10, knowledge of inverse proportions could be used to obtain a solution (C. Jiang, 2009). Although more students made errors on Problem 11 than Problem 10, the error pattern for Problem 10 was clearer than for Problem 11. Therefore, we chose Problem 10 as the example of this group of three problems.

The third group (Problems 3, 7, and 8) describes two motions of two objects. In Problems 3 and 7, the two objects are moving towards each other from two different points; in Problem 8, they are moving in the same direction with one ahead of the other. Problem 3 can be solved using arithmetic strategies; however, Problems 7 and 8 cannot if the student does not know the formulae. In Problem 7, the time taken for the two objects to meet is to be found; in Problem 8, the time taken for one to catch up with the other is to be found. A large number of the Chinese students used formulae to solve Problems 7 and 8. Therefore, fewer Chinese students made errors in solving the two problems than in solving Problem 3, as shown in Table 1. However, more Singapore students made errors in Problems 7 and 8 than in Problem 3. Students made similar errors in solving these two problems. Therefore, we chose Problem 7 as an example of this group.

The fourth group (Problems 2, 4, and 9) describes three motions of one object where the directions of the three motions can be the same. They also involved fractions to represent the relationships between distances of individual parts of the journey to the entire journey or to the remaining journey after the first motion. This kind of problem was found in a popular workbook written by Fong (1998). They were included to see whether the students could apply the concept of average speed from two motions to three motions. Indeed, we found that the students could (C. L. Jiang & Chua, 2010). Problem 4 is the only problem where fewer Singapore students made errors than their Chinese counterparts. It also provides opportunities to reveal the misconceptions students have with average speed, which appeared in all the three problems. Therefore, Problem 4 was chosen as an example of this group.

Table 3 shows the percentages of students in each group who made specific types of errors in solving the four problems. Across the four problems, more students made E1 errors in Problems 7 and 4. Therefore, in the following discussion of second level errors, we shall use students' errors in Problems 7 and 4 as the examples for E1 errors. Similarly, for the discussion of E2 errors, we will choose Problem 7 as the example. For the discussion of E3 errors, we shall choose both Problem 10 and Problem 4 as the examples. For the discussion of E4 errors, we shall choose Problems 6, 10, and 7 as the examples because more than one-third of the students made E4 errors on these three problems in either one or both countries. As E5 is "No solution", there is no need for extensive discussion.

Table 3
Number of Chinese and Singapore Students Making Different Types of Errors Solving the Four Problems

	E1	E2	E3	E4	E5
Problem 6					
Chinese	30	15	21	72	136
Singapore	7	6	11	319	148
Problem 10					
Chinese	10	12	96	320	158
Singapore	4	1	45	514	239
Problem 7					
Chinese	120	3	0	42	4
Singapore	67	27	65	427	154
Problem 4					
Chinese	40	13	369	35	98
Singapore	87	12	310	61	27

Note. E1 = Complete schema with errors; E2 = Incomplete schema with no errors; E3 = Incomplete schema with errors; E4 = Using irrelevant procedures; E5 = No solution.

Complete Schema with Errors – Errors in Calculation

In Problem 7, among the students making E1 errors, 52% of the Chinese (64/120) and 73% of the Singapore (49/67) students could produce the correct expression, $300 \div (84 + 60)$, but failed to get the correct answer of $2\frac{1}{12}$ hours or 2 hours and 5 minutes. Instead, their answers were $2\frac{1}{2}$ hours, $2\frac{1}{8}$ hours, 2 hours 15 minutes, $\frac{12}{25}$, and so on. Another 46% of the Chinese students (55/120) could set up the correct equation $84x + 60x = 300$, where x is the time taken for them to meet up, but they failed to solve it. Students frequently made errors in a variety of computation tasks (Ashlock, 1998), especially when the answers were not whole numbers.

In Problem 4, a typical correct solution was:

$1-\frac{1}{7}=\frac{6}{7}$, $\frac{6}{7}\times\frac{1}{3}=\frac{2}{7}$, $1-\frac{1}{7}-\frac{2}{7}=\frac{4}{7}$,

$72\times\frac{1}{2}$ = 36 km, $36\div\frac{4}{7}$ = 63 km,

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$, $63\div\frac{3}{2}$ = 42 km/h.

In the last two steps, addition and division of fractions are involved. Many students made errors in these steps. For example, 14% of the Singapore students (12/87) making E1 errors got a wrong result of 2 when computing ½ + ½ + ½. Among the students making E2 errors, 25% (10/40) of the Chinese and 54% (47/87) of the Singapore students made errors in computing $63\div\frac{3}{2}$; they produced results such as 24, 41, and 94.5, and so forth.

Incomplete Schema without Errors – No Errors at All

Among the students making E2 errors, 78% of the Singapore students (21/27) used guess-and-check methods for solving Problem 7 without making any errors, but could not reach the correct answer.

One typical solution was:

Time	1h	2h	3h	2h 30min	2h 15min
Distance covered by Mike (km)	84	168	252	210	189
Distance covered by Bill (km)	60	120	180	150	135
Distance covered together (km)	144	288	432	360	324
Check if they have travelled 300km	×	×	×	×	×

Incomplete Schema with Errors – Misunderstand Part of the Problem

In Problem 10, there is a hidden relationship, namely, the total distance is twice the distance of one way. Half of the Chinese (48/96) and 11% of the Singapore students (5/45) could successfully find the distance of one way. However, they took it as the total distance when finding the average speed for the whole journey, which was the most common error of E3 type. Twenty-seven percent (26/96) of the Chinese and 11% (5/45) of the Singapore students provided the following solution:

Let distance of one way be x km, then: $\frac{x}{40}+\frac{x}{120}=2$. Solving it, x=60km.

Therefore, average speed for the whole journey is 60÷2 = 30 km/h.

The doubling relationship between the distance of one way and the total distance did not seem to be apparent to some of the students.

In Problem 4, two fractions are given, the first one (1/7) is relative to distance of the whole journey, and the second one (1/3) is relative to the distance of the remaining journey after the first part. Among the students making E3 errors, 25% of the Chinese students (92/369) and

34% of the Singapore students (106/310) took the 1/3 as the relationship between the distance of the second part to the total distance. One typical solution was:

$1-\frac{1}{7}-\frac{1}{3}=\frac{11}{21}$, $72\times\frac{1}{2}$ = 36 km, $36\div\frac{11}{21}$ = $68\frac{8}{11}$ km, $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$, $68\frac{8}{11}\div\frac{3}{2}$ = $45\frac{9}{11}$ km/h.

Another common E3 error in Problem 4 was made in finding distances of the first two parts of the journey by setting up equations $\frac{1}{7}x\times\frac{1}{2}+(1-\frac{1}{7})\times\frac{1}{3}x\times\frac{1}{2}+72\times\frac{1}{2}=x$ where x is the total distance. Among the students making E3 errors, 8% of the Chinese students (30/369) were found to make this kind of mistake in finding the distance of the first and/or second part of the journey. They probably habitually used the formula D = “S”×T to find the distance of each part.

Problem 4 is also a kind of problem where three motions of one object were described to see whether students could apply the concept of average speed from two motions to three motions. Students’ incorrect responses revealed that they possessed two kinds of misconceptions about average speed. One was to take the average speed as the average of the individual speeds for the three parts of the journey. Among the students making E3 errors, 3% of the Chinese students (11/369) and 1% of the Singapore students (2/310) were found to hold this kind of misconception. One such solution given by a Chinese student is shown below:

$1-\frac{1}{7}-(1-\frac{1}{7})\times\frac{1}{3}=\frac{4}{7}$, $72\times\frac{1}{2}$ = 36 km, $36\div\frac{4}{7}$ = 63km,

$63\times\frac{1}{7}$=9km, $9\div\frac{1}{2}$=18km/h.

$63\times\frac{2}{7}$=18km, $18\div\frac{1}{2}$=36km/h.

(18+36+72)÷3 = 42 km/h.

The other misconception was to take the average speed as the total distance divided by the number of the parts of the journey. Among the students making E3 errors, 5% of the Chinese students (19/369) and 2% of the Singapore students (6/310) were found to hold this kind of misconception. One grade 6 Chinese student’s solution was:

$72\times\frac{1}{2}$ = 36 km, $36\div(1-\frac{1}{3})\div(1-\frac{1}{7})$= 63km, 63÷3=21 km/h.

For this problem, the times taken for the three parts are equal, and thus the average speed equals the average of the individual speeds. If the students did not give the reason for the use of this concept, they were considered to hold the first misconception. Furthermore, it takes more steps to find the speeds of the first two parts. More students were found to possess the second misconception of average speed. This might be because during the teaching of average speed more discussion about the first misconception was conducted so that such misunderstandings were eradicated. Those who used the second misconception seemed to use the general concept of “average” – sharing the total (distance) by the number of individuals (parts of the journey).

Using Irrelevant Procedures

As mentioned above, for the discussion of E4 errors, we shall focus on Problems 6, 10, and 7. In Problems 6 and 7, students seemed to use the formula T=D/S to find times. However, the distance and speed were often not matched correctly. In Problem 10 where the average speed for a round trip needed to be found, students revealed their misconceptions about average speed. Therefore, in the following we will discuss Problems 6 and 7 together, and Problem 10 on its own.

Using formulae versus sense making. Problem 6 gives the total distance (150 km), the total time (6 hours), and the speeds for the two parts of the journey (75 and 15 km/h), and asks students to find the time taken for the speed of 15 km/h (i.e., to solve the equation $15x + 75(6 - x) = 150$). We found that students who could not correctly answer it this way instead did 150÷6, which is to find the average speed for the whole journey, 150÷75 and/or 150÷15, which is the time taken to cover the whole journey at one of the speeds, but not 75÷15, which is the multiple relationship between the speeds. For example, among the students making E4 errors, 3% of the Chinese students (2/72) and 15% of the Singapore students (47/319) gave the answer 150÷6 = 25. However, the average speed they found was actually not useful. It was irrelevant because it was not required for solving this problem. Among the students making E4 errors, 3% of the Chinese students (2/72) and 4% of the Singapore students (14/319) gave the answer 150÷15; 1% of the Chinese students (1/72) and 3% of the Singapore students (10/319) provided the answer 150÷15 = 10, 10-6 = 4; and 7% of the Chinese students (5/72) and 7% of the Singapore students (22/319) gave the answer 150÷75 = 2, 6-2 = 4. These students seemed to apply the formula T=D/S to find times. However, they did not match them accordingly. They substituted the total distance and speed for one part of the journey into the formula instead.

Actually, for this problem, if we assume that Judy cycled for 6 hours (15×6=90km) and figure out the difference in distances covered (150-90=60km), by substituting the speed of the truck with the speed of cycling, one may be able to see the answer (that taking a lift in the truck for one hour makes up the difference). Students did not do it this way, probably because the question asked for the time she spent cycling. To keep a goal in mind while thinking about the problem in a different direction may have been too difficult for the students.

Similarly, in Problem 7, students seemed to use the formula T=D/S to find time. For example, among the students making E4 errors, 9% of the Singapore students (40/427) found the time taken by either Mike or Bill to cover the whole; 14% of the Chinese students (6/42) and 43% of the Singapore students (182/427) found the times taken by each to cover the whole alone. Among these, 17% of the Chinese students (1/6) and 32% of the Singapore students (58/182) showed no further work; 67% of the Chinese students (4/6) and 57% of the Singapore students (104/182) took the difference in times taken by each to cover the whole alone as the answer; 4% of the Singapore students (8/182) took the average of the times by each to cover the whole alone as the answer; and 17% of the Chinese students (1/6) and 7% of the Singapore students (12/182) took the sum of the times by each to cover the whole alone as the answer. Among the students making E4 errors, another 5% of the Singapore students (20/427) even assumed that they met at the midpoint of the way and took the time of either Mike or Bill as the answer or the sum of their times as the answer. And, 33% of the Chinese students (14/42) and 6% of the Singapore students (24/427) used the wrong formula by providing the solution 84-60 = 24 km/h, $300 \div 24$ = 12.5 hours.

Aside from the last response, the students who gave the above solutions seemed to use the formula T = D/S. However, they made incorrect assumptions such as one person making the entire journey for the meeting, or that the two would meet at the midpoint of the two places. After finding the times, they did the addition, subtraction, or found the average of the two times. These responses did not reveal any understanding that the two persons moved simultaneously at their own specific speeds until they met somewhere between the two places. Definitely, the two persons could not meet at the midpoint if their speeds were not equal.

Misconception of average speed: sum/difference of the speeds on the two ways. It is commonly recognized that students may take the average of individual speeds as average speed without realizing that the time durations for different speeds are different (Gorodetsky, Hoz, & Vinner, 1986; Thompson, 1994). This error was found in the solutions to Problems 5 and 11, and particularly to Problem 10. For example, in Problem 10, among the students making E4 errors, 53% of the Chinese students (168/320) and 50% of the Singapore students (258/514) gave the solution (40+120)/2=80. About 6% of the Chinese students (20/320) making E4 errors also held similar misconception, as they set up the equation $2x = 40+120$ where x is the average speed. 0.3% of the Chinese students (1/320) and 5% of the Singapore students (28/514) students took the sum of the two speeds as the average speed, and 0.3% of the Chinese (1/320) and 2% of the Singapore (12/514) students took the difference of the two speeds as the average speed.

Quite a few students' first two steps were 120+40 = 160 and 160×2 = 320. Perhaps they took the total time of two hours as the time for each way. Among the students making E4 errors, 3% of the Chinese students (8/320) and 2% of the Singapore students (10/514) wrote only those two steps; 12% of the Chinese students (38/320) and 11% of the Singapore students (56/514) provided the additional statement 320÷2= 160; 6% of the Chinese students (18/320) and 2% of the Singapore students (9/514) provided the additional statements 320÷2 = 160 and 160÷2 = 80. It is not clear what the twos meant in these solutions. Did they mean the total time or the number of parts of the journey? In the last solution, why did the students do the division twice? Did they try to get an answer between 40 and 120? Did they compensate for the lack of information (i.e., times on the two ways are unknown) by adding irrelevant data (i.e., the times on both ways are 2 hours) (Movshovitz-Hadar, Zaslavsky, & Inbar, 1987)? All of these possibilities call for further investigation.

Discussion

This error analysis indicated that many students had difficulties in solving multi-step word problems about speed. In some problems, more than half of the students from both countries could not reach the correct answer. More often, they made first-level E3, E4, and E5 errors. This indicates that they did not have a complete schema for solving the problems. In addition, Singapore students were more likely to make E4 errors than their Chinese counterparts.

The second-level error analysis indicates that there are quite a few similarities between the errors made by the Chinese and Singapore students. First, they often made errors in computations, especially when there were fractions involved as givens or answers. Second, the students who used guess-and-check strategies were more likely to make E2 errors. Many researchers have shown that guess-and-check can be a successful problem-solving strategy (Johanning, 2004; Kaur, 1998; Kilpatrick, 1967; Loh, 1991). However, when a guess-and-check strategy is used, the student needs to go through several rounds of the guess-and-check cycle, which is time-consuming. When the computations become more complicated, the student may not be able to persevere as expected. Among the eight standards for mathematical practice in Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), the first is to make sense of problems and persevere in solving them. In our results for Problem 7, it was clear that many of the students understood it well and made some effort in solving it. However, they could not persevere to reach the correct answer.

Third, in solving multi-step problems, it is difficult for students to understand the relationships clearly. This is especially notable for the hidden relationship between the total distance and the distance of one way for a round trip (or the equal relationship between the distances of the two ways of a round trip) as in Problem 10, and for the issue of correctly identifying the whole to which fractions refer as in Problem 4. This raises the question of whether the cognitive load is too heavy for the students, and if so, how we can help relieve it for them.

Fourth, a large number of students from grades 6-8 use irrelevant procedures in solving algebraic word problems. Researchers have investigated the undesirable (Sowder, 1988) or superficial coping strategies used by students (Verschaffel & De Corte, 1997). However, this study seems to suggest that students tried to use formulae such as T=D/S to find times, but failed to match appropriate distances and speeds to use the formula correctly. Did these students blindly operate on the given numbers or even try to look at cues such as distance and speed for a "reasonable" manipulation. Further investigation is needed.

Fifth, the second level error analysis also revealed that students have several misconceptions about average speed. Students used the misconception of average speed as the average of individual speeds in solving Problem 10 where a round trip is described. This kind of misconception is well known to researchers (Gorodetsky et al., 1986; Thompson, 1994) and is still very common among the secondary school students involved in this study. However, this study also seems to reveal that students hold another misconception of average speed as the sum of the speeds on the two ways. This misconception is rarely mentioned in the literature, but it was found to be prevalent in this study. For example, in solving Problem 3, three Chinese and 80 Singapore students provided a solution like 36 – 24 = 12 to find the speed on the way back home when being given 24km/h as the speed to the destination and 36km/h as the average speed (C. Jiang, 2005). These students probably oversimplified the concept of average as the sum of several numbers from the computational algorithm "add-them-all-up-and-divide" for computing the average (Cai, 2000b). Average speed is different from the general meaning of average in statistics. It is also different from the sum of individual speeds. The teaching of the average speed concept needs to discuss the different meanings of average and the differences in algorithms to find average and average speed. Being exposed to the concept of average speed in a broader sense may help students understand the concept better. Further experiments to probe this aspect need to be carried out.

Finally, the students seemed to have two more misconceptions of average speed. One is to find it through dividing the total distance by the number of the parts of the journey. The other is to take the difference of speeds as the average speed for a round trip. These kinds of misconceptions also need to be taken into account in the teaching and learning of word problems about speed.

References

Ashlock, R. B. (1998). *Error patterns in computation*. NJ: Prentice-Hall, Inc.

Babbitt, B. C. (1990) *Error patterns in problem solving*. Paper presented at the International Conference of the Council for Learning Disabilities (12th, Austin, TX, October 6, 1990). (ERIC No. ED338500).

Beaton, A., Mullis, I., Martin, M., Gonzalez, E., Kelly, D. & Smith, T. (1996). *Mathematics achievement in the middle school years: IEA's Third International Mathematics and Science Study (TIMSS)*, Boston College, Chestnut Hill MA.

Bell, A., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: The effects of number size, problem structure and context. *Educational Studies in Mathematics, 15*(2), 129-147.

Booth, L. R. (1983). *Misconceptions leading to errors in elementary algebra (generalized arithmetic)*. Unpublished doctoral dissertation, Shelsea College, University of London.

Bowers, J. S., & Nickerson, S. N. (2000) Students' changing views of rates and graphs when working with a simulation microworld. *Focus on Learning Problems in Mathematics, 22*(3-4), 10-25.

Cai, J. (2000a). Mathematical thinking involved in U.S. and Chinese students' solving process-constrained and process-open problems. *Mathematical Thinking and Learning*, *2*(4), 309–340.

Cai, J. (2000b). Understanding and representing the arithmetic averaging algorithm: an analysis and comparison of U.S. and Chinese students' responses. *International Journal of Mathematical Education in Science and Technology, 31*(6), 839–855.

Cai, J. (2004). Why do U.S. and Chinese students think differently in mathematical problem solving? Impact of early algebra learning and teachers' beliefs. *Journal of Mathematical Behavior, 23*(2), 135-167.

Cai, J., Lin, F. L., & Fan, L. (2004). How do Chinese learn mathematics? Some evidence-based insights and needed directions. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspective from insiders* (pp.535-554). World Scientific Publishing Co. Pte. Ltd.

Casey, D.P. (1978). Failing students: A strategy of error analysis. In P. Costella (ed.), *Aspect of Motivation* (pp.295-306). Melbourne: Mathematical Association of Victoria.

Clements, M. A. (1980). Analysing children's errors on written mathematical tasks. *Educational Studies in Mathematics, 11*(1), 1-21.

Cox, L. S. (1975). Diagnosing and remediating systematic errors in addition and subtraction computations. *The Arithmetic Teacher*, *22*(2), 151-157.

Engelhardt, J. M. (1977). Analysis of children's computational errors: A qualitative approach. *British Journal of Educational Psychology, 47*(2), 149-154.

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education, 16*(1), 3-17.

Fong, H.K. (1995). Schematic model for categorising children's errors in mathematics. *Hiroshima Journal of Mathematics Education, 3,* 15-30.

Fong, H. K. (1998). *Solving challenging mathematical problems: The heuristic approach for primary schools*. Singapore: Kingsford Educational Services.

Gorodetsky, M., Hoz, R., & Vinner, S. (1986). Hierarchical solution models of speed problems. *Science Education, 70*(5), 565-82.

Greer, B. (1992). Multiplication and division as models of situations. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp.276-295). New York: Macmillan.

Jiang, C. (2005). *Strategies and errors in solving speed problems: A comparative study between Chinese and Singapore students*. Unpublished PhD dissertation, National Institute of Education, Nanyang Technological University, Singapore.

Jiang, C. (2009). Word problems on speed: Students' strategies and errors. In K.Y. Wong, P.Y. Lee, B. Kaur, P.Y. Foong, & S.F. Ng (Eds.), *Mathematics Education: The Singapore Journey* (pp.244-262). Singapore: World Scientific.

Jiang, C.L., & Chua, B.L. (2010). Strategies for solving three fraction-related word problems on speed: A comparative study between Chinese and Singapore students. *International Journal of Science and Mathematics Education, 2010*(8), 73-96.

Jiang, L. (1998a). *Jiunian yiwu jiaoyu liunianzhi xiaoxue jiaokeshu: shiyan shuxue (11). [Nine-year Compulsory Education Six-year Primary Textbook Series: Experimental Mathematics 6A]*. Wuhan, China: Hubei Science and Technology Press.

Jiang, L. (1998b). *Jiunian yiwu jiaoyu liunianzhi xiaoxue jiaokeshu (12). [Nine-year Compulsory Education Six-year Primary Textbook Series: Experimental Mathematics 6B]*. Wuhan, China: Hubei Science and Technology Press.

Johanning, D. I. (2004). Supporting the development of algebraic thinking in middle school: A closer look at students' informal strategies. *Journal of Mathematical Behavior, 23*(4), 371-388.

Kilpatrick, J. (1967). *Analyzing the solution of word problems in mathematics, An exploratory study*. Unpublished doctoral dissertation, Stanford University.

Kaur, B. (1995). *An investigation of children's knowledge and strategies in mathematical problem solving*. Unpublished doctoral dissertation, Monash University, Australia.

Kaur, B. (1998). Mathematical problem solving in Singapore schools. *Teaching & Learning, 19*(1), 67-78.

Kaur, B., & Pereira-Mendoza, L. (1999). Singapore primary school TIMSS data: Whole numbers, fractions and proportionality. *The Mathematics Educator, 4*(1), 52-69.

Loh, W. F. (1991). *Secondary school students' understanding of algebraic expressions and equations*. Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Mayer, R. E. (1981). Frequency norms and structural analysis of algebra word problems into families, categories, and templates. *Instructional Science, 10*(2), 135-175.

Ministry of Education (MOE) (Singapore). (2000a). *Primary mathematics 6A* (3rd ed.). Ministry of Education (Singapore): Curriculum Planning and Development Division.

Ministry of Education (MOE) (Singapore). (2000b). *Primary mathematics 6B* (3rd ed.). Ministry of Education (Singapore): Curriculum Planning and Development Division.

Movshovitz-Hadar, N., Zaslavsky, O., & Inbar, S. (1987). An empirical classification model for errors in high school mathematics. *Journal for Research in Mathematics Education, 18*(1), 3-14.

Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1997). *Mathematics achievement in the primary school years: IEA's Third International Mathematics and Science Study (TIMSS).* TIMSS International Study Center, Boston College, Chestnut Hill, MA, USA.

Mullis, I.V.S., Martin, M.O., & Foy, P. (2008). *TIMSS 2007 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: Boston College.

Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., & Chrostowski, S.J. (2004). *TIMSS 2003 international mathematics report: Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades*. Chestnut Hill, MA: Boston College.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., et al. (2000). *TIMSS 1999: Findings from IEA's repeat of the third international mathematics and science study at the eighth grade*, *International mathematics report*. Chestnut Hill, MA: Boston College.

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common core state standards for mathematics*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Newman, M.A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. In M.A. Clements & J. Foyster (Eds.), *Research in Mathematical Education in Australia* (Vol.1) (pp.239-258). Melbourne: Swinburne Press.

Nichols, J. D. (1996). Reflections: A lesson in related rates. *Mathematics Teacher*; *89*(5), 390-93.

OECD. (2010). *PISA 2009 results: Executive summary.*

Olivier, A. (1989). *Handling pupil's misconceptions*. Presidential address delivered at the Thirteen National Convention on Mathematics, Physical Science and Biology Education, Pretoria, 3-7 July, 1989. Retrieved November 29, 2004, from http://academic.sun.ac.za/mathed/HED/Misconceptions.pdf

People's Education Press (PEP) (China). (1992). *Jiunian yiwu jiaoyu sannianzhi chuji zhongxue jiaokeshu: Daishu, Diyice (1). [The nine-year compulsory education three-year secondary school textbook series: Algebra 1A]*. PEP: Department of Junior Middle School Mathematics.

People's Education Press (PEP) (China). (1993a). *Jiunian yiwu jiaoyu sannianzhi chuji zhongxue jiaokeshu: Daishu, Diyice (2). [The nine-year compulsory education three-year* *secondary school textbook series: Algebra 1B]*. PEP: Department of Junior Middle School Mathematics.

People's Education Press (PEP) (China). (1993b). *Jiunian yiwu jiaoyu sannianzhi chuji zhongxue jiaokeshu: Daishu, Dierce. [The nine-year compulsory education three-year secondary school textbook series: Algebra 2]*. PEP: Department of Junior Middle School Mathematics.

People's Education Press (PEP) (China). (1994). *Jiunian yiwu jiaoyu liunianzhi xiaoxue jiaokeshu: Shuxue (No.5) [The nine-year compulsory education six-year primary school textbook series: Mathematics (No. 5)]*. PEP: Department of Primary Mathematics.

Radatz, H. (1979). Error analysis in mathematics education. *Journal for Research in Mathematics Education*, *10*(3), 163-172.

Radatz, H. (1980). Students' errors in the mathematical learning process: A survey. *For the Learning of Mathematics*, *1*(1), 16-20.

Roberts, G. H. (1968). The failure strategies of third grade arithmetic pupils. *The Arithmetic Teacher*, *15*(5), 442-446.

Robitaille, D. F., & Travers, K. J. (1992). International studies in achievement in mathematics. In D. A. Grouws (Ed.), *Handbook for research on mathematics and teaching* (pp.687-709). NY: Macmillan.

Sowder, L. (1988). Children's solutions of story problems. *Journal of Mathematical Behavior, 7*(3), 227-238.

Teh, K. L., & Looi, C. K. (2002a). *New syllabus D mathematics 1* (5th ed.). Singapore: Shing Lee Publishers Pte Ltd.

Teh, K. L., & Looi, C. K. (2002b). *New syllabus D mathematics 2* (5th ed.). Singapore: Shing Lee Publishers Pte Ltd.

Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp.179-234). Albany: State University of New York Press.

Tylee, J. L. (1997). Application: Computing airplane stopping distance: Applications of derivatives. *Mathematics Teacher, 90*(5), 398-402.

Verschaffel, L., & De Corte, E. (1997). Word problems: A vehicle for promoting authentic mathematical understanding and problem solving in primary school? In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp.69-97). UK: Psychology Press Ltd., Publishers.

Yuen, C. L. (1995). *Strategies and errors of children in solving fraction problems*. Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Zhang, D. (1998). *A guide to the research on mathematics education*. China: Jiangsu Education Publishing House.

Acknowledgements

This work was sponsored by the Scientific Research Foundation for Returned Overseas Chinese Scholars, State Education Ministry and China Postdoctoral Science Foundation, and a Research Grant from University of Macau. Any opinions expressed herein are those of the author and do not necessarily represent the views of the foundations. An earlier version of this paper was presented in Topic Study Group 15 at the 12th International Congress on Mathematical Education, July 8-15, 2012, Seoul, Korea. This paper was finalized while the author was visiting the University of Delaware and special thanks go to Professor Jinfa Cai for providing insightful comments on earlier versions of this manuscript.

Appendix A

Problem 6

On Sunday, Judy went to see her grandma who lives 150 km away. After cycling at an average speed of 15 km/h for a few hours, she got tired and took a lift from a passing truck. The truck's average travelling speed is 75 km/h. When she got to her grandma's house, she checked the time and knew that the trip took her 6 hours. Find the time she cycled.

Problem 10

Sunday morning, Rebecca and her parents went out to enjoy the natural scenery. On the way to the destination, they travelled at a slow speed of 40 km/h. On the way back, they drove at a faster speed of 120 km/h. When they came back home, they found that they had been out for 2 hours. Find the average speed for this round trip (ignoring time at the destination).

Problem 7

Two places R and S are 300 km apart. Mike left R and drove at 84 km/h towards S. At the same time, Bill left S at 60 km/h and drove towards R. How long did they take to meet?

Problem 4

Mike made a journey from City P to City Q. In the first half an hour, he covered $\frac{1}{7}$ of it. In the second half an hour he covered $\frac{1}{3}$ of the remaining journey. Finally he took another half an hour to finish the journey at a speed of 72 km/h. Calculate his average speed for the whole journey.