A Comparative Study on Pedagogical Content Knowledge of Mathematics Teachers in China and the United States

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Abstract

Using an interview protocol comprised of algebraic questions, U.S. and Chinese middle level teachers' responses were analyzed for differences in pedagogical content knowledge. Four themes were extracted from the analysis: concrete models and simple steps; practical and theoretical approaches; the application of cross-multiplication; generalization of problem and solution types. Our findings revealed U.S. teachers were more likely to use concrete models and practical approaches in problem-solving and promoting students' knowledge skills. However, they seemed to lack deep understanding of mathematical concepts as well as interconnections between concepts. Chinese teachers were inclined to utilize theories and procedures in teaching. By generalizing rules and strategies, Chinese teachers tended to integrate conceptual knowledge points as a conceptual network that made the knowledge applicable within multiple situations.

To meet the goal of competitiveness in mathematics education globally, international comparison studies (ICS) have received increasing attention for the purpose of sharing, discussing, and debating important issues across countries (Robitaille & Travers, 1992). Mathematics education in the United States has benefitted from the findings of ICS that result in initiatives towards improving students' mathematical performance. For example, the report of the Trends in International Mathematics and Science Study (TIMSS) has shown that both fourth and eighth graders' mathematics scores in the United States have made a significant increase in their 2007 averages when compared to their 1995 scores over the 12-year period (NCES, 2007). Although it is difficult to pinpoint the reasons for the improvement, it is plausible to give credit to focused, cross-nation comparisons, which resulted in educational policy changes, curriculum modification, and the development of teaching practices.

Meanwhile, the international comparative studies trigger our interest in delving into the underlying causes for performance differences between the United States and top-performing countries and districts, such as China-Taipei, Korea, Japan, and Hong Kong (NCES, 1999, 2003, 2007). A variety of research has been processed to identify the essential factors contributing to students' mathematical performance.

Teachers, as one of the most significant factors in mathematics education, do not only influence students on their content knowledge, but also play a critical role in shaping their misconceptions and confusions. However, large-scale investigation in terms of how teachers impact students' academic achievement is still sparse due to the difficulty of implementing among teachers extensively. Hence, small, in-depth studies become especially important and practical in examining the effects of teachers on students' mathematics learning. In this study, we attempted to use small-group comparisons to examine the characteristics of mathematics teachers' content knowledge and their teaching strategies as two essential components of pedagogical content knowledge (PCK) when teaching algebra. We hope the study will enrich our understanding of teachers' PCK and its impact on the effectiveness of mathematics education.

Theoretical Framework

Although a series of continual large-scale studies have shown U.S. students making significant progress in the international mathematics tests over the last decade (NCES, 2007), the status quo of U.S. students' continuous underperformance as opposed to their eastern Asian peers draws attention and speculation. Current studies mainly focus on identifying the distinctions for students' achievement within various content and competence domains between the top-performing Asian countries and the United States. Specially, Chinese students display superiority over their U.S. peers on base-ten counting (Miller & Stigler, 1987), computation and mental mathematics (Cai, 1997; Geary, Bow-Thomas, Fan, & Sigler, 1993), simple problem solving (Cai, 1995), and representational competence (Brenner, Herman, Ho, & Zimmer, 1999). A number of factors have been suggested as potential contributors for the divergence in specific mathematical areas, such as students' beliefs, attitudes, motivations of mathematics learning, teachers' instructional strategies, and focus of school curriculum, etc. (Chen & Stevenson, 1995; Wang & Lin, 2005).

Undoubtedly, teachers are considered as one of the most significant factors that affect student learning in mathematics due to the critical role teachers playing in the teaching and learning process. According to the National Council of Teachers of Mathematics (2000), "Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies" (p. 17). Teachers' mathematics knowledge is essential to effective teaching and student learning (Ball & Bass, 2001; Shulman, 1987). To teach effectively, teachers must possess the knowledge and skills that consists of (a) general ways to present content to students; (b) understanding of students' common conceptions, misconceptions, and difficulties when encountering particular situations; and (c) specific teaching strategies that can be used to meet students' diverse learning needs, which derives from Shulman's original notion of *pedagogical content knowledge (PCK)* (Rowan, Schilling, Ball, & Miller, 2001; Shulman, 1987). A wealth of studies has elaborated on the definition of PCK in accordance with particular attributes and needs within diverse disciplines. For instance, in the domain of mathematics education, PCK has been defined as involving three components: knowledge of content, knowledge of curriculum, and knowledge of teaching as shown in figure 1 (An, Kulm, & Wu, 2004). These three components interconnect and interact intensely in a complex way. Accordingly, profound understanding of mathematical content is likely to reciprocally interact with the effectiveness of teaching (An et al., 2004).

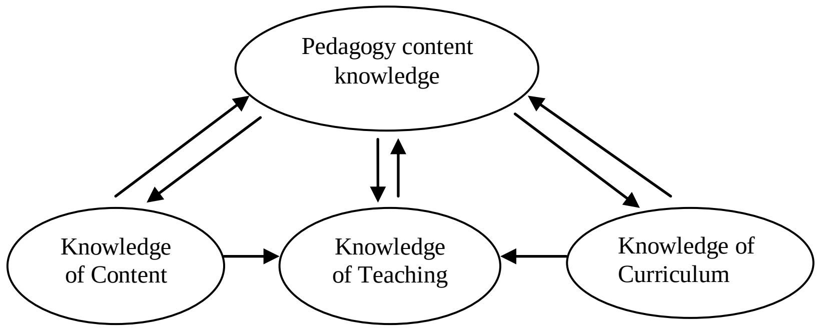


Figure 1. The network of pedagogical content knowledge is adapted from An, Kulm, & Wu, 2004.

Furthermore, effectiveness of teaching is also affected by teachers' epistemological belief on the learning process, that is, learning as knowing or learning as understanding (An et al., 2004). Teachers who hold the belief of "learning as knowing" usually focus on infusing students with specific concepts or procedures without identifying their misconceptions and understanding level. In contrast, teachers with the "learning as understanding" belief tend to encourage students to internalize their newly acquired concepts through the process of integrating prior and current knowledge as a netted whole. As a result, academic outcomes under the epistemological belief of learning as knowing are classified at a surface level while those under the belief of learning as understanding at a mastery level.

Numerous research has been carried out to compare Chinese and American teachers' knowledge of content (e.g., Ma, 1999) and knowledge of effective teaching (e.g., An et al., 2004), but little research examines teachers' pedagogical content knowledge by integrating these two crucial components of PCK. Our current study focused on investigating U.S. and Chinese teachers' PCK differences by comparing their problem-solving strategies and teaching methods in well-chosen algebraic areas. Our research question was: *What are the differences in pedagogical content knowledge between Chinese and American teachers when observing their problem-solving processes in specific algebraic areas?*

Methodology

Participants of the study were four teachers from the west Texas area of the United States and four teachers from one school in a large city of the Sichuan province in southwest China. In addition, the teachers were teaching at the same middle school level for the same subject area of mathematics in schools with similar enrollment of 500 students. However, there were demographic differences between these two school districts. The U.S. teachers were from a school district with ethnic composition of 49.7% Hispanic, 35.4% Caucasian, and 14.9% African American. On the contrary, all Chinese teachers were from one district with dominantly more than 95% students from one ethnic group – Han.

A set of eight algebraic word problem questions was given to participating teachers to solve within an interview session. All the questions were developed with the purpose of comparing and contrasting the use of different problem solving solutions and teaching strategies between teachers from two countries on topics of symbols, surface area, proportional reasoning, and patterns (see Figure 2). The questionnaire was prepared identically in both English and Chinese versions.

Teacher participants were encouraged to "think aloud," a research method that reveals people's thinking process through language, when attempting the problems. They were also asked to illustrate their teaching strategies if the questions were used in their mathematics courses. Each participant was interviewed and videotaped individually. In addition, their dialogues to themselves as well as to the interviewers during the problem solving processes were transcribed to analyze for similarities and discrepancies in teachers' content knowledge and teaching strategy aspects of PCK. The lead author conducted all interviews with the Chinese teachers and translated them into English. The second author and two colleagues in the mathematics education program conducted interviews with the U.S. teachers. Each interview lasted approximately 20 to 30 minutes.

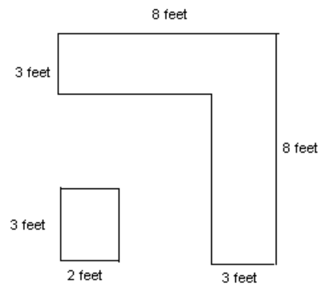
Incorporating an interview method of data collection assists researchers to better understand teachers' knowledge of content and teaching strategies. The qualitative approach tends to be more insightful than the frequently utilized quantitative approach, as indicated in the Board on International Comparative Studies in Education:

> There is a great need for small, in-depth studies of local situations that permit cross-cultural comparisons capable of identifying the myriad of causal variables that are not recognized in large-scale surveys…much survey data would remain difficult to interpret and explain without the deep understanding of society that other kinds of studies provide…research in cross-national contexts benefits from increased documentation of related contextual information, it would be useful to combine large-scale surveys and qualitative methods. (Gilford, 1999, p. 22)

Data collected from the interviews were coded using norms modified from An et al.'s (2004) coding categories (see Table 1). By using constant comparative data analyses (Glaser & Strauss, 1967), all categories were grouped and thereby formed four themes within the domains of content knowledge and knowledge of teaching strategies.

1. On Friday the low temperature in Nome, Alaska, was -6ºF, and the high temperature was 14 ºF. How much warmer was the high temperature than the low temperature?

2. Mr. Jones wants to install new countertops on his two kitchen counters. The drawing below shows the dimensions of the counters. What is the least amount of material needed to cover the tops of both kitchen counters?\



3. A software company employs 450 workers. It plans to increase its workforce by eight employees per month until it has doubled in size. Write an equation that can be used to determine m, the number of months it will take for the company's workforce to double in size and solve this equation.

4. Larry's favorite painting has a width of 30 inches and a height of 24 inches. Larry had a reduced copy of the painting made as a gift for his father. If the reduced picture of the painting was similar to the original painting and the height of the reduced picture was seven inches, what was the width of the reduced picture?

5. Raymond packs boxes for an appliance company. He can pack a large box in 10 minutes and a small box in four minutes. He needs to pack 10 large boxes and 20 small boxes. If 2.5 hours remain before closing time, will Raymond have time to finish the work before closing time if he works without stopping?

6. The table below shows a relationship between x and y. Write an equation that best represents this relationship?

x	y
0	3
1	8
3	18
4	23
6	33

7. Sharon played an electronic game. There were 15 questions, of which she answered three incorrectly. At this rate, how many questions should Sharon expect to answer in correctly if she answers a total of 135 questions?

8. To make a certain shade of orange paint, Calvin must add 20 ounces of yellow paint to every 50 ounces of red paint. If he uses 200 ounces of red paint, what is the number of ounces of yellow paint he should add to get the shade of orange he wants?

Figure 2. Algebraic Ideas Assessment

TABLE 1

Categories for describing teachers' problem-solving and teaching strategies in dealing with Algebraic Ideas Assessment (adapted and modified from An et. al., 2004)

Category	Brief definition
1.	Prior knowledge: Know students' prior knowledge and connect it to new knowledge.
2.	Concept or definition: Use concept or definition to promote understanding.
3.	Rule and procedure: Focus on rule and procedure to reinforce the knowledge.
4.	Draw picture or table: Use picture or table to show a mathematical idea.
5.	Give example: Address a mathematical idea through examples.
6.	Provide students an opportunity to think and respond: Promote students to think problems and give them chances to answer questions.
7.	Manipulative activity: Provide hands-on activities for students to learn mathematics.
8.	Attempts to address students' misconceptions: Identify students' misconceptions.
9.	Use questions or tasks to help students' progress in their ideas: Pose questions or provide activities to increase the level of understanding for students.
10.	Provide activities and examples that focus on student thinking: Create activities and examples that encourage students to ponder questions.
11.	Use one representation to illustrate concepts: Apply repeated addition to address the meaning of fraction multiplication, or use area to address the geometrical meaning of fraction multiplication.
12.	Using more than one representation to illustrate fraction multiplication: Apply both repeated addition and area to address the meaning of fraction multiplication.
13.	Unintelligible response: Provide response that is not relevant to the question.
14.	Incorrect: Provide a wrong answer.

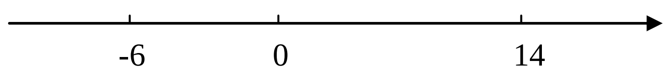
Results

By comparing the teachers' problem-solving strategies and their self-revealing cognitive processes via the "thinking aloud" technique when solving the eight algebra word problems, we found that both the U.S. and Chinese teachers had extensive content knowledge backgrounds regarding algebraic topics of proportion, rate, equation with variables, and linear functions. In addition, both groups were equipped with essential skills to teach students the content via various approaches. However, major differences were found in their content knowledge as well as knowledge of teaching strategies between the U.S. teachers and their Chinese counterparts. Four themes were extracted from the differences within the two components of PCK. For Content Knowledge, differences between U.S. and Chinese mathematics teachers were manifested in three themes of *concrete models and simple steps, practical and theoretical approaches,* and *application of cross-multiplication;* For knowledge of Effective Teaching, differences between the two groups of teachers can be found in the theme of *generalization of problem and solution types.*

Content Knowledge

Theme 1 - Concrete models and steps. When attempting question 1, finding the difference between two temperatures with opposite signs, all four U.S. teachers drew a number line in order to make the difference between the temperatures visual whereas all Chinese teachers preferred to use a simple calculation of subtracting the lower temperature from the higher temperature (see Figure 3).

U.S. teachers: (step 1)



(step 2) 6 + 14 = 20

Chinese teachers: 14 – (-6) = 20

Figure 3. Different solutions by American and Chinese teachers

The U.S. teachers rationalized that using a visualized representation made problem solving easy for students since all they needed to do was to find the distance between two points on the number line. Using concrete or graphic representations has been supported by NCTM (2000) due to its potential to develop meaningful understanding of mathematical concepts. With the number line, the U.S. teachers tended to find the distance from -6ºF to 0ºF (6 degrees) and the distance from 0ºF to14ºF (14 degrees), then add the two distances to get the correct answer of 20 degrees (see Figure 3).

In contrast, Chinese teachers tended to solve this problem by using simple computation steps instead of drawing any explicit graphics. In the Chinese teacher group, one participant explained her reason of discouraging drawing since generating a graphical representation was time-consuming and distracting. The other two explained that students were allowed to use number lines only at the beginning of learning negative numbers, which helped visualize the relationships between numbers with opposite signs. Once the students were familiar with algorithms of integers, they were no longer encouraged to use number lines and teachers expected them to be able to visualize the number line in their mind rather than on the paper. In other words, Chinese teachers were likely to impose higher expectations on students to develop mental visualization skills in middle school. In addition, Chinese teachers expressed that students' accumulative exercises in mental visualization would benefit their abilities with spatial visualization and abstract thinking in their future coursework of advanced algebra and geometry.

By comparing their solutions, evidence was found that American teachers were more likely to relate problems to concrete situations for the purpose of visualizing the relationship between numbers whereas Chinese teachers tended to identify the mathematical relationships by looking for key words. For instance, in question 1, all Chinese teachers came to the same solution of subtracting the lowest degree (-6ºF) from the highest degree (14ºF). In their minds, the word *warmer* in the problem implied the solution as a simple subtraction of the smaller number from the larger number. Their problem-solving method was encased with simplicity and efficiency, which was consistent with their teaching strategy. As An et al. (2004) asserted, "the main characteristic of Chinese mathematics is the development and practice of *accurate* and *efficient* means of computation and to apply these in real life" (p. 160).

Theme 2 - Practical and theoretical approaches. Participants' responses indicated that the U.S. teachers were more likely to solve problems with practical and specific approaches whereas Chinese teachers tended to utilize theories or generalized strategies. For instance, given question 6 asking to develop a linear equation based on data of two variables provided in a table, three out of four American teachers adopted the strategy of a "guess and plug in" brute force method. In contrast, Chinese teachers demonstrated varied approaches rather than simply testing all the numbers in possible relationships. For example, after reading the problem, a Chinese teacher instantly laid out all algebraic possibilities of equations satisfying positive relationships between variables x and y. He also explained pertinent attributes for each equation form in order to identify the appropriate pattern between variables for this question.

> There are several types of functions fit for this kind of relationship: the linear function, the proportional function, the quadratic function, the exponential function, and the logarithmic function. Besides the increasing relationship, we should look at the relationships of the variable changes too. Viewing from the angle of parameters, (*this question*) when x increases 1 unit, y increases 5 units; when x increases 2 units, y increases 10 units. This phenomenon tells us that x and y are proportional in relationship and they should be expressed as a linear function….If the growth becomes faster and faster, it is possible to be an exponential function; if the growth gets more and more slowly, it's likely to be a logarithmic function. (Chinese teacher 1)

This reasoning-and-proving method had been adopted primarily by Chinese teachers in practice since it not only ensures the accuracy of problem solving, but also broadens students' mathematical knowledge into an integral structure. Through analyzing diverse possibilities and then sifting to the correct answer, the teacher simplified a complicated problem to a multiple-choice type of problem, that is, to select a correct response among several options. Nevertheless, this teaching strategy demands a solid understanding of underlying theories of numerous concepts and procedures, sometimes far beyond what is required to solve the problem at hand.

Besides eliminating incorrect options with their extensive knowledge in mathematics, Chinese teachers were inclined to use strategies of generalization. To solve the same question, Chinese participant 2 generalized two solutions by using a linear function and coordinate systems.

> Like I just wrote, the tendency of changes in x's is: the second x value is one more than the first one; and the third x value is two more than the second one. It can be found that with the change of x's, the value of y changes correspondingly, that is, with every unit of increase of x is a constant amount of change in y. Therefore, students may conjecture that this is a linear function. Another way we teach students to solve this problem is to use a coordinate system. We usually teach these two methods. (Chinese teacher 2)

In contrast, three out of four U.S. teachers did not show evidence that they had any strategy to employ other than "guess and plug in." Below is a typical dialogue of the way a U.S. teacher explained how the problem should be solved.

> They would look to see if there is a pattern in the differences.…One-five, two-ten, one-five, two-ten (*points to left column for the first number and right column for second*)….When x is 0, y is 3. So x + y = 3 and that might be one check that they (students) do, which is 3 (writes equation). But 1 + 8 is 9, so they would back away from that and look for another relationship to what is going on here. (U.S. teacher 4)

U.S. participant 4's explanation showed that she did not see the pattern for the changes of x and y based on the information from the table. Although she found that x and y intervals were aligned with a pattern of "one-five, two-ten, one-five, two tens," she still could not figure out a constant relationship between them as 'x times 5 plus 3 equals to y." After the interviewer hinted about drawing a line, she finally figured out a linear relationship between x and y. This problem-solving episode implied that U.S. teachers may have not possessed a conceptual understanding of linear functions. Such difficulties experienced by the U.S. teachers would most likely impact their students' learning, interest, and achievement in mathematics.

Theme 3 - The application of cross-multiplication. For those questions involving proportional relationship (Questions 4, 7, and 8), both U.S. and Chinese teachers adopted various approaches when setting up proportional relationships and using a cross-multiplication strategy in solving problems. U.S. teachers manifested divergent attitudes toward using cross multiplication in solving proportion problems. For example, when asked about the application of cross-multiplication, two U.S. teachers expressed frustration and confusion. One teacher stressed that cross-multiplication never made sense to students since it was merely taught as a procedure or as a "short-cut" without helping students to understand why it works. Another teacher could not interpret cross-multiplication correctly regarding why the method works. Nevertheless, the other two U.S. teachers embraced the use of cross-multiplication while they emphasized how this strategy should be taught to facilitate student understanding of the method. See excerpt below.

> Initially when they're being taught this method, they know an equation is about balance between the sides of an equation. And in order to get rid of this denominator you have to multiply both sides by 50 (*denominator*)…eventually they would shorten it because they see you get rid of it (*denominator*) here…I teach them to understand the process….(U.S. teacher 4)

Reversely, Chinese teachers used cross-multiplication (without doubt or hesitation) as if it was one of the most basic algorithmic rules. Due to the emphasis placed on efficiency and accuracy in the Chinese examination system, cross-multiplication has been viewed not only as a procedure but a basic concept needed to be grasped. Generally, Chinese teachers value both conceptual understanding and procedural proficiency equally. Particularly, they believed that procedural proficiency is built upon students' solid understanding of mathematics, which also positively impacts the extent of knowledge mastery inversely. Our finding was consistent with a previous study (An et al., 2004), which reported that far fewer U.S. teachers believed that using procedures and rules were effective in building mathematical ideas than their Chinese peers.

Knowledge of Effective Teaching

Theme 4 - Generalization of problem and solution types. With regard to teaching methods, Chinese teachers were more likely to generalize a problem in terms of different situations and then identified the appropriate solution in response. For example, to solve problems involving decimals (question 3), a Chinese teacher summarized three different types of solutions to round a decimal into an integer and asserted that students should learn to read the problem carefully and find out the key words to match the corresponding situation. He said,

> Three methods for it: rounding-up, taking-out, and adding-one methods. This problem needs to use "adding-one" method. Since the result is 56.2, we cannot keep it as 56 months, (*because*) not enough, we should "add one" to 57 months. Another dimension of the meaning in this question is that varied conditions determine dissimilar solutions. (Chinese teacher 1)

By generalizing problems, Chinese teachers tried to reinforce the belief of mathematics as a netted concept web within a learner's mind. In this way, students are able to connect new concepts to their prior knowledge spontaneously as well as to search for solutions from varied angles. For instance, when dealing with the pattern problem (question 6), Chinese teacher 2 described two distinct solutions, "because of the interval as y, we can solve this problem with two ways: the first, visual observation… the second way (is) that students probably think about is the application of function …"

In response to question 4

involving two similar figures, another Chinese teacher not only reviewed a variety of ratios, but also illustrated their relationships. He stated,

> Students need to make sense of one principle: there are different ratios for two similar figures, such as similar ratio, area ratio, and volume ratio. In terms of two specific shapes, the area ratio equals to the square of the similar ratio; the volume ratio equals to the cube of the similar ratio….therefore, identifying what belongs to a similar ratio is crucial, such as the ratio of height, length, median line, perimeter, and so on. All the mentioned ratios are called similar ratio because they are units of length. (Chinese teacher 4)

In contrast, U.S. teachers were more likely to use one method in solving the particular problem scenario without illustrating any larger holistic "big idea" or relating to parallel situations. For example, when solving the same problem 4, a U.S. teacher said,

> The height of reduced painting was 7 inches, so, what was the width of reduced picture? OK, whenever you have similar figures…you have similar figures the dimensions, or let's see the scale factor. I guess you can set up proportion. That is what I am trying to say. It is 30 inches as the width (writing), and the height is 24 inches in your original. And the smaller one, the height is 7 inches. We are looking for the width of the reduced picture. You can set up a proportion, you can see 30 inches is to 24 inches, and unknown width, is to 7 inches. And shortcut is to cross-multiple. (U.S. teacher 2)

Apart from divergent teaching strategies in problem solving depicted above, U.S. and Chinese teachers showed different attitudes and perspectives in teaching it. As mentioned previously in theme 2 of *Content Knowledge*, some U.S. teachers were inclined to use and teach it just as a "short-cut" for the only purpose of simplifying the problem-solving processes in questions. On the contrary, all the Chinese teachers espoused the necessity of procedural practices; they believed that developing students' proficient procedural skills helps to reinforce what they had learned and allowed them to transfer skills easily to new knowledge to novel problem situations (An et al., 2004).

Discussion and Conclusion

Because of the "gatekeeper" status of algebra to advanced mathematics study as well as its significance penetrating K-12 curriculum (Davis, 1985; Oliver, Izsak, & Blanton, 2002), this study used a set of algebraic questions to compare and contrast middle school teachers' relevant content knowledge and knowledge of teaching as two essential components of pedagogical content knowledge. Particularly, these teachers' problem-solving skills and teaching strategies were carefully examined.

Findings in this study may identify the factors that contribute to the discrepancy in mathematics achievement between American and Chinese students that caused by teacher impact. Teachers' pedagogical content knowledge is essential for effective teaching which directly affects students' learning outcomes. In this study, little evidence revealed an obvious discrepancy in teachers' content knowledge in algebra. Nonetheless, significant differences were noticed in their problem solving strategies and teaching methods. Chinese teachers preferred to tackle problems by looking for key words in order to set up direct relationships while U.S. teachers considered drawing visual representations to be helpful when solving problems. Furthermore, Chinese teachers were likely to sort problems into categories based on wording structure and seek diverse approaches to deal with whereas American teachers were in favor of taking a practical, brute force approach, such as 'trial and error'.

All these phenomena mirrored disparate beliefs and values in two education systems: Chinese teachers treat *accuracy* and *efficiency* as the primary goal for solving problems as well as teaching mathematics while only *accuracy* is emphasized in U.S. (NCTM, 2000). Accordingly, forming a conceptual understanding within a netted knowledge structure becomes the prerequisite to reach this goal. In particular, Chinese teachers were accustomed to teaching students by constantly linking mathematical concepts, which allowed students to review and reinforce concepts and procedures from time to time. Consequently, students are likely convinced that mathematics is a well-structured body of knowledge. In contrast, U.S. teachers heavily relied on practical approaches and external aids, such as graphic representation and visual manipulatives. Despite its advocacy of various forms of representations, NCTM (2000) warned that such representations and manipulatives sometimes are used as if they end in themselves. In other words, if graphic representations are not used to reach in-depth understanding, students are likely to end up with "learning as knowing" instead of "learning as understanding."

The results of this study support an idea that teaching for understanding is the key for successful math education. Procedural learning can become valuable only when it is based on students' understanding the underlying mathematical logic and reasoning. Chinese teachers value procedural proficiency as an equal weight of conceptual understanding. In their perspectives, procedural proficiency not only results from a genuine mastery of knowledge but also resonates conceptual understanding to some degree. In contrast, U.S. teachers have more ambivalent attitudes toward procedural learning, such as the attitudes expressed by U.S. participants in use of cross multiplication. Some U.S. teachers opposed cross multiplication as conceptual learning since it is merely a shortcut in calculation. In fact, if students have conceptual understanding of fractions, they are capable to deduce why this shortcut works all the time. Using a shortcut should be built on the foundation of understanding.

Evidently, a series of schooling and non-schooling factors should be taken into consideration when exploring teachers' impact on students. For instance, researchers have found that Asian American students are good and even better performers in mathematics when they were exposed to the same curriculum and teaching practices along with other American ethnic peers, which suggests that different educational systems between east and west cannot exclusively explain the significant academic gap internationally. Therefore, non-schooling factors, such as differences in culture and language, should also be taken into consideration in terms of their effects on teaching-and-learning process. There is no doubt that teaching is a cultural activity. But in what form and to what extent culture influences the teaching practice as well as students' attitude, motivation, and performance are worthy of intensive investigation across countries.

In conclusion, the results of this study suggest that remarkable differences exist in the pedagogical content knowledge between U.S. and Chinese teachers, which may result in dissimilar teaching and learning outcomes. U.S. teachers were more likely to use concrete models and practical approaches in problem-solving and promote students' knowledge skills.

However, they seemed to lack deep understanding of underlying mathematical theories. The Chinese teachers were inclined to utilize theories and procedures for teaching and learning. By generalizing rules and strategies, they were able to integrate knowledge points as a whole network. As An et al. (2004) indicated, different education belief systems produce different attributes of pedagogical content knowledge. In order to really improve teaching, we should invest far more than we do now in generating and sharing knowledge about teaching (NCES, 2007).

Given the fact that this study included a small sample size, its findings cannot necessarily be generalized to all mathematics teachers in the United States and China. Especially, since this study heavily relied on qualitative methods, the findings may not be applied to the population of middle school mathematics teachers in these two countries. However, these findings do provide some insights on teachers' pedagogical content knowledge through the lens of an international comparative study.

In a future study, we will examine how cultural beliefs influence teachers' pedagogical content knowledge. To improve the levels of generalization and application, a mixed methods approach will be utilized in which both quantitative and qualitative data will be collected and analyzed.

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